Minsky from the bottom up - Formalising the two-price model of investment in a simple agent-based framework

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Abstract
Using a simple agent-based stock-flow consistent (AB-SFC) framework, this paper presents a fully formalised version of the two-price model of capital investment which forms an important but relatively neglected component of Hyman Minsky’s Financial Instability Hypothesis. The model presented in this paper consists of an agent-based sector of consumption goods firms incorporating behavioural rules formalising the two-price model as well as three strongly simplified aggregated sectors. The model is calibrated empirically using moments drawn from US data, demonstrating that it is capable of producing empirically plausible time-series. Simulations show that the uncoordinated investment strategies of individual firms lead to the emergence of investment-driven cycles at the aggregate level. Desired investment is centrally driven by firms’ expectations, with complementarities in firms’ investment strategies together with changes in financing conditions leading to the emergence of cyclical dynamics. It is also shown that the introduction of a stylised public sector can contribute to a stabilisation of the model.

Keywords: Stock-flow consistency, Agent-based models, Financial instability, Investment
JEL-Classification: E12, E22, E32, C63

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1 Introduction

Since the mid-1980s there has been a steadily growing literature seeking to capture the central notions of the economic ideas of Hyman Minsky, in particular his financial instability hypothesis (FIH; e.g. Minsky, 1977, 1978, 1986), in formal mathematical models. This literature has grown even more rapidly following the global financial crisis and by now there exists an enormous range of models with a Minskyan flavour, using diverse methodologies and theoretical foundations, and emphasising diverse aspects of Minsky’s work. There is however relatively little literature building directly on one element of Minsky’s work which is closely related to the FIH - namely the two-price model of investment as presented, for instance in *John Maynard Keynes* (Minsky, 1975) and *Stabilizing an Unstable Economy* (Minsky, 1986). There exist some papers incorporating core ideas from the two-price model in a simplified form to model firms’ investment behaviour. However, the two-price model has to my knowledge never been fully formalised at the microeconomic level and subsequently embedded in a macroeconomic model. Using a simple agent-based stock-flow consistent (AB-SFC) framework, this paper sets out to do precisely that.

The model presented in this paper depicts three aggregated sectors modelled using a pure SFC methodology, namely households, banks and capital goods firms, along with an agent-based sector of consumption goods firms. While the aggregated sectors are modelled in a strongly simplified fashion, close attention is paid to the modelling of the agent-based consumption goods firms. The resulting model is calibrated empirically using the a set of moments drawn from US-data aimed at summarising the aggregate results of firm behaviour, demonstrating that in this respect it is capable of producing plausible macroeconomic dynamics. The baseline simulation is characterised investment- and finance-driven cycles which emerge endogenously from the uncoordinated microeconomic investment behaviour of firms. Subsequent simulation experiments focus on the process firms use to gauge the prospective yield on capital goods, along with somewhat stylised versions of monetary and fiscal policy. An introduction of a simple form of sentiment dynamics, strengthening strategic complementaries in the investment decision, significantly exacer-
bates macroeconomic fluctuations, while the introduction of a share of ‘fundamentalist’ firms has a stabilising effect. These experiments highlight the importance of expectations and opinion formation in macroeconomic dynamics, an aspect which has not always featured prominently in the literature on Minsky models. Further experiments show that policy interventions can turn out to be highly effective in dampening the fluctuations produced by the baseline model. Fiscal policy interventions produce a significant reduction in macroeconomic volatility. However, an equivalent reduction in volatility can also be obtained through the use of conventional monetary policy. This is surprising insofar as interest rate policy has not played a prominent role in the Minskyan literature. Overall, the paper shows that the relatively neglected two-price model of investment forms a suitable basis for the modelling of Minskyan dynamics, leading to results at the macroeconomic level which are in line with those outlined by Minsky using a more narrative methodology. It demonstrates how macroeconomic real-financial cycles can emerge from uncoordinated investment strategies of individual firms at the microeconomic level, contributing to the relatively small literature of heterogeneous agent Minsky-models. Moreover, the empirical calibration of the model represents an innovation to the Minskyan literature which has to date been dominated by purely theoretical models.

The paper is structured as follows. Section 2 provides a brief overview of the literature on Minsky models, including existing works incorporating the two-price model. Section 3 outlines the model. Section 4 describes the calibration procedure and presents the baseline simulation while section 5 contains some simulation experiments. Section 6 concludes. Parameter and initial values used in simulations can be found in the Appendix.

2 Minskyan dynamics and the two-price model in the literature

Nikolaidi and Stockhammer (2017) provide a comprehensive survey of the literature on Minsky models, categorising existing papers into one group focusing on debt and interest dynamics with little emphasis on asset prices, and another in which asset prices play
the key role. Within the first category, they distinguish between a range of sub-classes pertaining to various characteristics of the surveyed models,\footnote{e.g. Kaleckian or Kaldorian goods markets, the presence of credit rationing, or endogenous target debt ratios etc.} surveying literature ranging from older, pre-financial crisis contributions such as Foley (1987) and Keen (1995) all the way to recent papers such as Nishi (2012) and Dafermos (2018). Their second category contains the existing papers drawing on the two-price model.

Minsky’s own most detailed discussions of the two-price model of capital investment can be found in Minsky (1975) and Minsky (1986), the former of which contains figure 1. Minsky argues that there are two distinct prices of capital goods, namely a demand price ($P_k$ in the figure) and a supply price ($P_l$ in the figure), the terminology being taken from Keynes (1936). The demand price is a function of the prospective yield on existing capital goods, whereas the supply price denotes the cost of production and acquisition (financing cost) of an additional unit of a capital good.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{two-price-model.png}
\caption{The two-price model of investment; taken from Minsky (1975, p.106)}
\end{figure}

Minsky argues that firms will invest until the demand and supply prices are equal, at first
using internal funds and then resorting to external finance.\(^2\) As soon as internal funds are exhausted, the demand price will begin to fall away from \(P_k\) with rising investment due to an increased perception of risk on the part of borrowers. The supply price will increase as investment beyond internal funds will incur a financing cost which, due to increased perception of risk on the part of lenders, will rise as investment increases further.\(^3\) Possible increases in internal funding as a result of aggregate increases in investment, together with loosening financing conditions and declining margins of safety (changing the slopes of the curves depicting borrowers’ and lenders’ risk) can lead to a self-reinforcing upward trend in investment which is eventually reversed when expectations about yield are no longer confirmed by actual outcomes.\(^4\)

Among the models classified into the second category proposed by Nikolaidi and Stockhammer (2017), Delli Gatti and Gallegati (1990) draw most closely on the two-price model, using it to justify the formulation of an aggregate investment function. Delli Gatti et al. (1994a) and Delli Gatti et al. (1994b) - the latter of which includes Minsky himself as a co-author - build on the aforementioned model and include similar formulations for the aggregate investment function. An aggregate investment function along the same lines can also be found in Nasica and Raybaut (2005), and the formalisation of investment demand advanced by Taylor and O’Connell (1985) is also motivated by the two-price model. All of these models, alongside the vast majority of those surveyed by Nikolaidi and Stockhammer (2017) are macrodynamic models without an explicit microeconomic dimension.

Michell (2014) constructs a ‘hybrid’ AB-SFC model in which only firms are agent-based, and shows a way to reconcile Minsky’s FIH with the arguments put forward by Lavoie and Seccareccia (2001), according to whom Minsky’s theory contains a ‘missing link’ between the microeconomic investment behaviour of firms and aggregate dynamics of leverage and

\(^2\)The two-price model hence bears clear resemblance to Tobin’s model of investment based on the q-ratio (Tobin, 1969). However, as pointed out by Wray and Tymoigne (2009), there are important differences related to the role of financing conditions and uncertainty/risk assessment.

\(^3\)These notions are based on the principle of increasing risk as advanced by Kalecki (1971).

\(^4\)The issue of what exactly brings about the turning point in the investment boom is somewhat controversial, see e.g. Lavoie (2014, Ch. 4) and Toporowski (2005, Ch. 14).
financial fragility. The model presented by Pedrosa and Lang (2018) proceeds along similar lines to address the critique of Lavoie and Seccareccia (2001). Both papers emphasise the importance of the distribution of profits and debt among firms for systemic financial fragility. The purpose of the present paper is not to contribute to this particular debate. Instead, I am principally interested in examining the dynamics which emerge if agent-based firms behave according to the two-price model of investment, an assumption which is not incorporated in either Michell (2014) or Pedrosa and Lang (2018). Nevertheless these works show the usefulness of an AB-SFC approach in examining aspects of Minsky’s theory and their hybrid structure makes them methodologically very close to my own model.

In a series of papers, Chiarella and Di Guilmi (2011, 2012a,b, 2014, 2017) present a family of simple Minsky models building closely on the approach of Taylor and O’Connell (1985) but containing an agent-based firm sector. Similarly to the latter paper, the investment behaviour of firms in these models is based on what can be considered a simplified form of the two-price model, whereby investment is a function of a ‘shadow price’ of capital which depends on animal spirits (derived from a stylised stock market) and the interest rate. By contrast, my model does not depict stock market dynamics but instead directly makes the demand-price of capital a function of the sentiments prevailing among firms investing in capital goods, in particular their expectations about future earnings, which appears more closely in line with what is described by (Minsky, 1975, Ch. 5). The contribution of the present study relative to the aforementioned papers is that instead of providing a simplified and reduced formulation, this paper contains a full and detailed dynamic formalisation of the two-price model very closely in line with the arguments advanced by Minsky, and shows that these assumptions can indeed give rise to aggregate Minsky-cycles. In doing so, the paper contributes to the literature on heterogeneous agent Minsky-models which, as noted by Nikolaïdi and Stockhammer (2017), is somewhat underdeveloped. Moreover, the focus on expectation and opinion dynamics, the importance of which is emphasised in the simulation experiments described below, is relatively underdeveloped in the existing Minskyan literature. Finally, as noted by (Nikolaïdi and Stockhammer, 2017, p. 1327),
the Minskyan literature is dominated by theoretical models and “the vast majority of the authors make no attempt to estimate econometrically the key equations of their models or to calibrate the models in order to produce the patterns observed in the real data”. There exists a small literature of econometric papers testing particular aspects of Minsky’s FIH (e.g. Greenwood-Nimmo and Tarassow, 2013; Nishi, 2019), but there is no systematic connection between these and the theoretical models advanced by various authors. In this sense, the empirical calibration procedure applied in this paper represents an innovation to the literature, showing that the proposed model is capable of producing empirically plausible dynamics.

3 Model outline

Tables 1 and 2 provide an overview of the aggregate sectoral structure and the accounting relationships depicted in the model.

Table 1: Balance Sheet Matrix

<table>
<thead>
<tr>
<th>Households</th>
<th>C-firms</th>
<th>K-firms</th>
<th>Banks</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Deposits</td>
<td>+$D_h$</td>
<td>+$D_{fc}$</td>
<td>−$D$</td>
<td>0</td>
</tr>
<tr>
<td>Loans</td>
<td>−$L$</td>
<td>+$L$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Firms’ Equity</td>
<td>+$E_f$</td>
<td>−$E_{fc}$</td>
<td>−$E_{fk}$</td>
<td>0</td>
</tr>
<tr>
<td>Banks’ Equity</td>
<td>+$E_b$</td>
<td>−$E_b$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>+$K$</td>
<td></td>
<td>$K$</td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td>$V_h$</td>
<td>$V_{fc}$</td>
<td>$V_{fk}$</td>
<td>$V_b$</td>
</tr>
</tbody>
</table>
Table 2: Transactions Flow Matrix

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>C-firms</th>
<th>K-firms</th>
<th>Banks</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>( -C )</td>
<td>+C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>( -I )</td>
<td>+I</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wages</td>
<td>( +W )</td>
<td>( -W_c )</td>
<td>( -W_k )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Dividends</td>
<td>( +Div )</td>
<td>( -Div_{fc} )</td>
<td>( -Div_{fk} )</td>
<td>( -Div_b )</td>
<td>0</td>
</tr>
<tr>
<td>Interest Loans</td>
<td>( -iL )</td>
<td>+iL</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Saving</td>
<td>( (Sav_h) )</td>
<td>( (Sav_{fc}) )</td>
<td>( (Sav_{fk}) )</td>
<td>( (Sav_b) )</td>
<td>0</td>
</tr>
<tr>
<td>Δ Deposits</td>
<td>( -\Delta D_h )</td>
<td>( -\Delta D_{fc} )</td>
<td>+( \Delta D )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Δ Loans</td>
<td>+( \Delta L )</td>
<td>-( \Delta L )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Σ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

As can be seen, the model consists of four sectors, namely households, capital goods firms (K-firms), banks and consumption goods firms (C-firms), of which only the latter is modelled using an agent-based methodology whilst the rest remains aggregated (in particular, the C-firm sector is assumed to consist of 50 firms). Households hold bank deposits and own the firms and banks. They receive all wage and dividend incomes and have expenditures on consumption goods. Consumption goods firms produce consumption goods and invest in capital goods, with part of this investment being financed by loans from the banking sector. They also hold some bank deposits as a buffer. Capital goods firms produce capital goods but hold no assets or liabilities. Banks give loans and receive deposits. The economy depicted here is a stationary one in that labour and capital productivities are constant and there is no long-term growth.

### 3.1 Sequence of events

Over the course of one simulation period, the following sequence of steps takes place:

1. C-firms calculate capital depreciation and loan payments due at the end of the period. They set their price levels for the current period. An aggregate consumption price index is calculated and the price of capital goods is set equal to the latter.
2. Households form expectations about their income and wealth and set their consumption budget based on this. Consumption demand is then distributed between C-firms who produce on demand.

3. Based on the results of the consumption process, C-firms form their expectations about the prospective yield on capital goods, determine their internal financing and consequently formulate their investment demand and the implied demand for loans. Capital goods are produced on demand.

4. C-firms calculate their profits and dividend payments for the period. Bankrupted C-firms are replaced according to the procedure outlined below.

5. Banks and K-firms determine their profits and dividend payments. All remaining accounting identities are calculated and checked for stock-flow consistency.

The remainder of the section outlines the behavioural assumptions of the model sector by sector.

### 3.2 Households

In order to focus on the behaviour of the agent-based C-firms, all other sectors are deliberately kept as simple as possible. As such, the household sector only contains few behavioural assumptions. Households formulate their demand for consumption goods according to

\[
    c^d = \frac{\alpha_1 YD_{-1} + \alpha_2 V_{h,-1}}{CPI},
\]

where CPI is an aggregate price index for consumption goods, YD is disposable income and \( V_h \) is household wealth. This is a consumption function which is very commonly used in the literature on SFC models (see Godley and Lavoie, 2007), and also frequently appears in this or roughly equivalent forms in the agent-based literature (Dawid and Delli Gatti, 2018). It states that households wish to consume a fixed fraction of their disposable income as well as a fixed fraction of their accumulated wealth. As documented
by Dawid and Delli Gatti (2018), various ABM frameworks endogenise the consumption propensities, for instance to implement a buffer-stock saving rule. I keep the consumption behaviour intentionally as simple as possible and hence stick to constant parameter values. Households are assumed to supply any amount of labour demanded by firms at the fixed wage rate $w$. Any financial savings are accumulated in the form of unremunerated bank deposits.

### 3.3 K-firms

Capital firms use a labour-only technology (specifically $y_k = N_k$ where $N_k$ is labour employed by K-firms) to produce capital goods on demand, which are sold at a constant price $p_k$. They distribute all profits to households such that their equity remains at its initial value, set here to 0.\(^5\)

### 3.4 Banks

Banks take unremunerated deposits from C-firms and households. They make loans for capital investment to C-firms, supplying any amount demanded at the endogenously determined loan rates outlined below. As such this model does not depict any form of quantity rationing on the credit market which, while important to Minskyan theory in general, is not the object of analysis here. In addition, banks are implicitly assumed to finance, free of interest, on-demand production of consumption and capital goods, with the consequence that firms are never liquidity-constrained in their production decisions. Due to the instantaneous nature of production and sale in this model, this lending of ‘working capital’ does not show up on any balance sheet as production loans are instantaneously repaid. The banks distribute all profits to households, possible losses due to firm defaults are absorbed by their equity.

The banking sector in this model may also be regarded as an amalgamation of banks, the government and the central bank as the ‘fiscal’ and ‘monetary’ policy measures simulated

\(^5\)K-firms cannot make losses since the constant $p_k$ is set to a value greater than the constant unit labour cost.
in section 5 are implemented through that sector. In line with this interpretation, it is assumed that the banking sector does not go bankrupt if its equity goes negative. This is reasonable insofar as the unitary banking/central banking/government sector is the creator of the sole means of payment in the system which by assumption is always universally accepted, such that this sector can always make good on any promises to pay. The choice to model the banking sector as an aggregate which does not go bankrupt of course prevents me from depicting any dynamics related bank failures etc. and their feedbacks on the real economy, which also play an important role in Minskyan theory. However, the goal of this paper is to focus on the modelling of the firm sector and in particular to investigate the macroeconomic implications of implementing the two-price model of investment at the level of the agent-based C-firms.

3.5 C-firms

The capital stock held by C-firms depreciates by a fixed rate $\delta$ in every period. In each period, C-firms also pay interest on their outstanding loans and repay a fraction $\theta$ of the principal. Each consumption goods firm sets its price as a mark-up over unit labour cost, $w$ (labour productivity being normalised to 1). In each period, a random subset of firms is allowed to change its mark-up rates such that on average each firm can change its mark-up every four periods (one simulation period representing one quarter). If a firm is allowed to change the mark-up, the latter is set according to

\begin{equation}
\Omega^j_c = \frac{2\Omega}{1 + \exp \left( \varepsilon \left( \frac{s^j_{t-1}}{1/nF} - 1 \right) \right)},
\end{equation}

with $\varepsilon < 0$, where $\Omega$ is some ‘normal’/conventional mark-up which anchors the price-level, $s^j_{t-1}$ is a four-period moving average of firm $j$’s market share and $nF$ is the number of C-firms. The rule hence causes firms with a higher market share to increase their mark-up while firms with a relatively lower share will decrease their mark-up. This rule is somewhat similar to the one used by Pedrosa and Lang (2018) as well as Dosi et al. (2010) and in this form represents a simple way to introduce some degree of price-competition into the
model whilst ensuring that firms’ prices remain relatively similar. Due to the behavioural assumption of mark-up pricing and since equation 2 precludes negative mark-ups, firms never set a price below unit labour cost. To distribute households’ aggregated demand for consumption goods between firms, two indicators of relative price and size of each firm are calculated as follows:

\[
\hat{p}_c^j = \left(\frac{p_c^j}{\bar{p}_c}\right)^{\iota_1}, \\
\hat{k}^j = \left(\frac{k^j}{\bar{k}}\right)^{\iota_2},
\]

where \(\bar{p}_c\) and \(\bar{k}\) are the average price and capital stock, with \(\iota_1 < 0\) and \(\iota_2 > 0\). Both indicators are then normalised. Firm \(j\)’s share is then calculated as

\[
s^j = \lambda_1 s_{j-1}^j + (1 - \lambda_1) \frac{\hat{p}_c^j + \hat{k}^j}{2},
\]

which is then multiplied by an autocorrelated, normally distributed shock with mean 1 and standard deviation \(s^j\sigma\) and normalised once more to ensure that \(\sum_{j=1}^{n_F} s^j = 1\). A firm with a lower price will hence tend to attract a greater share of demand, as is the case e.g. in Pedrosa and Lang (2018) or the K+S model (Dosi et al., 2010). The relative capital stock is included to reflect the idea that larger firms, regardless of their price, will tend to attract more demand, including because they are more likely to be able to fulfil it due to having higher productive capacity (cf. Michell, 2014; Pedrosa and Lang, 2018). Having received their shares of the aggregate demand for consumption goods, C-firms produce according to

\[
y_c^j = \min(c_d^j, \kappa k_{j-1}^j) \\
N_c^j = y_c^j,
\]

paying wages \(W_c = wN_c^j\) to their workers and giving rise to a net cash flow before loan payments which is given by \(CF = y_c^j p_c^j - W_c\).\(^6\) At this point, in order to enable an im-

\(^6\)C-firms may receive demand which exceeds their productive capacity \(\kappa k_{j-1}^j\), in which case consum-
plementation of the two-price model, firms must calculate a measure of available internal financing which can enter into the determination of the demand-price of capital. It is thus assumed that prior to the investment decision, firms decide on a desired level of dividends as a fraction $\gamma^j$ of their cash flow. Beyond being convenient for the implementation of the two-price model, there is some empirical support for a non-residual dividend policy which is largely independent of the firm’s investment strategy, although overall the evidence does appear mixed (Partington, 1985; Baker, 2009; Michaely and Roberts, 2012). $\gamma^j$ is endogenised as

$$\gamma^j = \frac{2\gamma}{1 + \exp (\tau (\pi^j - \pi))},$$

with $\pi^j$ being the firm’s annualised profit rate prior to investment and depreciation and $\pi$ being a fixed, ‘normal’ level of this profit rate. An endogenisation of the retention rate as a function of profitability both seems empirically reasonable (Jensen et al., 1992; Benartzi et al., 1997; Charles, 2008) and has been used in the literature on Minsky models (Yilmaz and Stockhammer, 2019). Internal financing is then determined as

$$\text{int}^j = \max(0, (1 - \gamma^j)CF^j - iL^j - rep_{L}^j),$$

where $iL^j$ and $rep_{L}^j$ are loan interest payments and principal repayments respectively. While C-firms do hold bank deposits, it is assumed that they do not use these to finance capital investment. Rather they are designed as a buffer stock to cover unexpected short-falls in demand, to enable the firm to continue loan and dividend payments. In the next step, C-firms determine the prospective yield of capital goods. In this baseline version of the model, C-firms are assumed to simply extrapolate the current yield over the lifetime of their stock of capital goods, giving rise to

$$Q^j = \frac{c_{d}p_{c}^j - W_{c}^j}{r_{d} + \delta},$$

ation is rationed (this effect is very slight in all simulations) and ex-post shares are rescaled to reflect this.
where \( r_d \) is a fixed discount rate which is identical between firms. This formulation appears very much in line with what is described by Minsky (1975), including in that it uses the yield before interest or dividend payments as a basis for calculation (see Minsky, 1975, p. 106). This provides all the ingredients necessary to formulate a function for the demand price of capital, i.e. the equivalent of the curve \( P_k \) in figure 1. I write this as

\[
(9) \quad p_{d,j}^{k,j} = \frac{Q_j - \mu_1^j (I_j)^2}{k^j},
\]

making the demand price of capital for firm \( j \) a decreasing function of firm \( j \)'s current investment. This formulation for the demand price of capital might still be regarded as akin to a \( q \)-type ratio as it is based on the discounted future earnings firms expect to make using their capital stock relative to the current value of the capital stock. However, as indicated above, share prices (which are not included in the model) do not enter into the determination of investment and indeed there appears to be only very limited empirical evidence for the relevance of the conventional \( q \)-ratio in explaining investment (López Bernardo et al., 2016). In the present model, investment is centrally driven by firms’ demand expectations, bringing it closer to (post-)Keynesian theories of demand-driven investment. The term \( \mu_1^j (I_j)^2 \) is intended to incorporate the notion of borrowers’ risk emphasised by Minsky, i.e. the idea that with increasing investment, the riskiness of investment perceived by the investor increases, thus decreasing the demand price. \( \mu_1^j \) is endogenised\(^7\) according to

\[
(10) \quad \mu_1^j = \frac{2\mu_1}{1 + \exp \left( - \left( \beta_1 l^j + \text{rep}^j - \beta_2 \pi^j \right) \right)},
\]

so that the curve representing the demand-price of capital is steeper for firms with a high debt service to cash flow ratio and flatter (leading cet. par. to higher investment) for firms with a high profit rate. Note that, in contrast to Minsky’s \( P_k \)-curve, my function

\(^7\)The endogenisation of \( \mu_1 \) is intended to capture, in a simple way, shifting attitudes towards risk and margins of safety on the borrowers’/investors’ side over the course of the cycle. The parameter \( \mu_2 \) introduced below could be similarly endogenised to capture the same notion for lenders. Under the parametrisation presented here this was found to make little difference to simulation results, but it may make for an interesting extension in a modified version of this model.
(depicted in figure 2) does not contain a discontinuity whereby the demand price only falls off after all internal funds have been invested as in figure 1. The reason for this is that figure 1, and more generally Minsky’s presentation of the two-price model, appears to imply that firms always either invest all available internal funds (and possibly more) or they invest nothing at all. That is, figure 1 does not permit the possibility of a case in which $0 < I^j < int^j$, whereas my formulation permits this case and hence appears more reasonable and general. In this case, instead of “borrowers’ risk” it thus also makes more sense to speak of “investors’ risk” as $p_d^{k,j}$ is decreasing even in internally financed investment. By contrast, as can also be seen in figure 2, my schedule for the supply price of capital does contain the discontinuity depicted by Minsky in figure 1, being given by

\[
p_s^{k,j} = \begin{cases} 
p_k & \text{if } I^j \leq int^j, \\
 p_k + \left(1 - \frac{int^j}{I^j}\right)r_Lj & \text{if } I^j > int^j,
\end{cases}
\]

where $r_Lj$ is the loan interest rate charged to firm $j$ so that $r_Lj$ represents the total interest cost of a unit of externally financed investment. The banking sector determines the interest rate charged to any firm $j$ as

\[
r_L^j = r_0 + \mu_2 \left( \frac{\hat{i}L^j + \hat{rep}^j_L}{CF^j} \right).
\]

The interest rate is hence an increasing function of the debt service to cash flow ratio, a measure of financial fragility often favoured by Minsky (see e.g. Minsky, 1986). The term after the constant $r_0$ in equation 12 is meant to capture the notion of lenders’ risk, i.e. that the financing cost of investment increases with the perceived riskiness for any investment exceeding internal funds. Importantly, $\hat{i}L^j$ and $\hat{rep}^j_L$ in equation 12 refer not to current loan payments (which are determined with certainty by past borrowing) but rather to future ones conditional on the loans being taken out in the current period, i.e. $\hat{i}L^j$ is a function of $r_L^j$ and $L^j$ (the stock of loans) after investment, and $\hat{rep}^j_L$ is a function of $L^j$ after investment. Equation 12 can be solved for $r_L^j$ and then substituted into equation 11 to obtain $p_s^{k,j}$ as a function of firm $j$’s investment only. Once the demand- and supply-
price schedules have been so determined, C-firms find the point at which the two curves intersect, invest accordingly and (if necessary) take out loans. The resulting amounts of investment and new loans simultaneously also determine $r^j_L$ according to equation 12.

![Figure 2: Demand and supply price of capital](image)

Towards the end of each period, C-firms calculate their profits and update their stock of deposits. Since bank deposits act as a buffer for firms, they formulate a target for their stock of deposits as a fraction $\zeta_1$ of current sales. If profits are positive and available deposits are sufficient to do so, firms pay the desired dividend formulated during the determination of current internal financing plus a fraction $\zeta_2$ of the deviation (which may be negative) of actual deposits from the target value. If profits are negative, C-firms do not pay a dividend. C-firms go bankrupt if either their equity becomes negative due to persistent losses or if they have insufficient liquidity to service their debt.\(^8\) In either case all their loans are written off, all their deposits are removed from the banks’ balance sheet, and a fraction $1 - \chi$ of their capital is scrapped while the rest is transferred to the banks. The losses to the banking sector from a firm default are hence $L^j - D^j_f - \chi K^j$.\(^9\) A new firm

---

\(^8\)Note that by assumption, C-firms can only borrow to finance capital investment. They are not permitted to borrow in order to cover principal or interest payments on loans or to finance dividend payouts. As such, the distinction between hedge, speculative and Ponzi firms often used by Minsky (e.g. Minsky, 1978) is not present in this model, though it would represent an interesting extension.

\(^9\)This expression may indeed be positive, in which case it is booked as a profit for the banking sector.
is then assumed to enter the market. This firm receives a transfer of deposits from the households sufficient to purchase the remaining capital stock previously transferred to the banks and to have an initial buffer of bank deposits. This way of modelling bankruptcy is very similar to the ones employed in Caiani et al. (2016) and Pedrosa and Lang (2018).

Due to the fashion in which consumption demand is distributed between C-firms, it also occasionally happens that a particular firm becomes extremely small relative to the rest. It does not necessarily go bankrupt in this case according to the conditions above and even if it does, this only leads to the capital stock and initial market share of the firm replacing it being even smaller. It is therefore assumed that if a C-firms’ market share remains below 0.0001 (i.e. 1/100th of a %) for more than four simulation periods, it exits the market whereupon its entire capital stock is scrapped, its deposits removed and its loans written off. It is then replaced by a new firm which purchases an initial capital stock (equal to the mean value of the other firms’ capital stocks) from the K-firms, financed by households, and also receives a transfer of deposits from the households.

4 Calibration & baseline simulation

The Appendix provides a full list of initial and parameter values which must be set prior to simulation of the model. In order to arrive at a satisfactory baseline calibration, I employ a simulated minimum distance estimator, the use of which in ABMs is discussed by Grazzini and Richiardi (2015).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\iota_1$</td>
<td>Price sensitivity of demand distribution</td>
<td>-3 : -0.5</td>
</tr>
<tr>
<td>$\iota_2$</td>
<td>Firm size sensitivity of demand distribution</td>
<td>0.5 : 1.5</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Persistence of demand distribution</td>
<td>0.5 : 0.95</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Persistence of demand distribution shock</td>
<td>0.25 : 0.95</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Strength of demand distribution shock</td>
<td>1 : 10</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Borrowers’ risk adjustment parameter</td>
<td>0.1 : 3</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Payout ratio intercept</td>
<td>0.3 : 0.5</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Price adjustment parameter</td>
<td>-5 : -1</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Payout ratio adjustment parameter</td>
<td>-70 : -30</td>
</tr>
</tbody>
</table>
In particular, I apply the method of simulated moments (see e.g. Gilli and Winkler, 2003; Franke and Westerhoff, 2012; Schmitt, 2018) to calibrate 9 of the model’s parameters by attempting to minimise the distance between a set of 9 moments/statistics generated by the model and their empirical counterparts. The list of parameters which are empirically calibrated in this manner is shown in table 3. For the empirical calibration procedure I obtain the following quarterly macroeconomic time-series for the United States for the period Q1 1952 to Q3 2018:

1. Real Gross Private Domestic Investment (Federal Reserve Economic Data, series identifier GPDIC1)
2. Gross Domestic Product (Federal Reserve Economic Data, series identifier NA000334Q)
3. Nonfinancial corporate business; net interest and miscellaneous payments (Federal Reserve Economic Data, series identifier BOGZ1FA106130003Q)
4. Nonfinancial corporate business; debt securities; liability (Federal Reserve Economic Data, series identifier NCBDBIQ027Q)

Following seasonal adjustment where required, series 2-4 are used to calculate the ratios of firms’ interest payments and debt to nominal GDP. To remove trend components from the empirical time series (real investment, corporate debt to GDP and corporate interest payments to GDP), I follow the approach recommended by Hamilton (2018), running the following regression model on all empirical time series (where $x$ denotes the time series):

\begin{equation}
\ln(x_t) = \alpha + \sum_{i=8}^{11} \beta_i \ln(x_{t-i}) + \epsilon_t.
\end{equation}

The residuals from this regression are interpreted as the cyclical component, which is used to calculate the moments/statistics used in the calibration procedure, while the predicted values represent the trend component. Using the de-trended series I calculate the following moments/statistics:

- The standard deviations of real investment, firm debt to GDP and firm interest payments to GDP ($sdI$, $sdDebt$ and $sdInt$)
- The one-lag autocorrelations of the aforementioned series ($acI$, $acDebt$ and $acInt$)
• The contemporaneous cross-correlation between real investment and firm debt to GDP as well as that between real investment and firm interest payments to GDP ($ccIDebt$ and $ccIInt$)

• a measure of cycle frequency, given by the average number of times the de-trended time series of real investment crosses the x-axis (i.e. changes sign) within 100 periods ($freq$)

As becomes clear from both the choice of parameters to be calibrated as well as the moments/statistics to be reproduced, the focus of this calibration procedure is on the behaviour of C-firms and the aggregate dynamics this gives rise to. The reason for this is that the rest of the model is extremely simplified and I would not expect the model to be capable of replicating a richer set of moments (indeed it is not constructed for the purpose of doing so). Denoting the vector of parameters to be calibrated by $\Theta$, the vector of empirical moments/statistics by $m^e$ and the vector of simulated moments/statistics by $m$, I aim to minimise the loss function

$$L(\Theta) = (m(\Theta) - m^e)'W(m(\Theta) - m^e),$$

i.e. I aim to find

$$\Theta^* = \arg\min_{\Theta} L(\Theta).$$

Following Franke and Westerhoff (2012), the weighting matrix $W$ is the inverse of the variance-covariance matrix of the empirical moments/statistics, obtained using bootstrapping. In calculating $L$, the variance of the chosen moments/statistics is hence taken into account. The calibration proceeds by sampling the parameter space delimited by the initial ranges shown in table 3 using latin hypercube sampling Salle and Yildizoglu (2014) and simulating the model 50 times with different, reproducible seeds for each sampled parameter combination. Sampling is then repeated around points of the parameter space which appear promising in terms of the value of the loss function, eventually increasing the number of Monte Carlo repetitions to 100 as the relevant parameter space becomes smaller. This procedure continues until further sampling no longer produces noticeable
reductions in the value of the loss function.\textsuperscript{10} The values of the parameters thus calibrated are given in the Appendix. Table 4 compares the moments/statistics produced by the calibrated model (calculated on %-deviations from the post-transient mean of the stationary simulated time series) to their empirical counterparts.

With the exception of the autocorrelations of debt to GDP and interest payments to GDP, the model is able to reproduce the observed statistics reasonably closely. This should be regarded as a satisfactory result in particular given the simplicity of the model beyond the fairly detailed C-firm sector. If the sector of C-firms modelled here were embedded in a more detailed and carefully specified macroeconomic framework, it should be possible to considerably improve the fit.

Table 4: Comparison of empirical & simulated moments/statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Empirical</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>sdI</td>
<td>0.12731</td>
<td>0.13887</td>
</tr>
<tr>
<td>sdDebt</td>
<td>0.05959</td>
<td>0.07052</td>
</tr>
<tr>
<td>sdInt</td>
<td>0.15041</td>
<td>0.16816</td>
</tr>
<tr>
<td>ac\textsuperscript{1}I</td>
<td>0.89259</td>
<td>0.94496</td>
</tr>
<tr>
<td>ac\textsuperscript{1}Debt</td>
<td>0.89060</td>
<td>0.98338</td>
</tr>
<tr>
<td>ac\textsuperscript{1}Int</td>
<td>0.89155</td>
<td>0.98725</td>
</tr>
<tr>
<td>ccIDebt</td>
<td>-0.40181</td>
<td>-0.59299</td>
</tr>
<tr>
<td>ccIInt</td>
<td>-0.30745</td>
<td>-0.29592</td>
</tr>
<tr>
<td>freq</td>
<td>9.72763</td>
<td>9.05996</td>
</tr>
</tbody>
</table>

The full initialisation protocol of the model proceeds by imposing an initial stationary state (in which all firms are identical) on the model.\textsuperscript{11} This is done by imposing restrictions on parameters and the initial values of stocks and flows until a sufficient number of degrees of freedom have been removed for all remaining initial values and some parameter values to be implied by these restrictions. For instance, through imposing an initial capital stock, the full-capacity income to capital ratio $\kappa$, the depreciation rate $\delta$ and an initial value for capacity utilisation, the initial values of aggregate consumption, investment, disposable

\textsuperscript{10}Of course, due to the large size and high dimensionality of the parameter space, it is impossible to determine whether the parameter combination produced by this procedure does in fact represent the global minimum of the loss function. The results obtained do however seem satisfactory.

\textsuperscript{11}This stationary state is a purely analytical device which conveniently ensures that all initial values are mutually consistent, making use of the SFC structure. It should not be interpreted as possessing any empirical relevance or being some sort of fundamental attractor, and indeed the model departs from this stationary state upon simulation with stochastic microeconomic shocks.
Income, GDP, wages and employment are implied by the assumption of a stationary state. Some parameters the values of which are set using the empirical calibration procedure outlined above are ‘free’ in the sense that their value does not have an impact on the values of other parameters or initial values in the initial stationary state, whereas others (such as $\gamma$) do feed into the determination of other parameters/initial values. For instance, to produce an initial stationary state, I need to impose, based on equation 10, that

$$
\beta_2 = \beta_1 \cdot \frac{r_L L + \theta L}{\pi CF}
$$

such that the values of $\beta_1$ and $\beta_2$ are consistent with a stationary state. Similarly the empirically calibrated value of $\gamma$ feeds into a range of other parameters as well as initial values, such as $\mu_2$ (the sensitivity of the lending rate to the debt service to cash flow ratio) and $r_L$ (the bank’s initial lending rate). Despite the empirical calibration of a subset of parameter values, I am still left with some degrees of freedom in initialising the model. Whenever this is the case, I choose empirically plausible values where possible. Tables 6 and 7 in the Appendix provide a full list of initial and parameter values. The calibrated model is simulated for a total of 2000 periods (quarters). Simulations are carried out for 100 Monte Carlo repetitions with different, reproducible seeds. Following the beginning of the simulation with stochastic elements, the model diverges from the initial stationary state and after a transition period settles into the dynamics described below. The plots below show the model behaviour from simulation period 500 onwards.\(^{12}\)

Figure 3 depicts the dynamics of real GDP and consumption for a single, representative run of the model. It can be seen that the model economy experiences persistent cycles in output. Figure 4 shows that investment leads the cycle whereas consumption in figure 3 roughly coincides with the dynamics of GDP, meaning that the cycles are indeed driven by fluctuations in investment.

\(^{12}\)All files necessary to repeat the simulations shown below can be found at https://github.com/SReiss1/Minsky.
Figure 3: Real GDP and consumption

Figure 4: Real GDP and investment
The left panel of figure 5 shows that the average leverage ratio of C-firms lags the cycle. The right panel shows that turning points in the debt service to cash flow ratio (which is an important input for C-firms’ behavioural equations) appear to immediately precede turning points in GDP. At the trough of the cycle, profitability eventually bottoms out and begins to rise, owing to firm defaults and falling interest rates, which leads to an upturn in the payout ratio $\gamma$ and (together with declining debt ratios) a decline in the borrowers’/investors’ risk parameter $\mu_1$. Investment and dividend payouts begin to rise, with rising payout ratios increasing the proportion of investment which is financed by debt. This leads to a boom which eventually comes to an end as firms become increasingly financially fragile and profitability fails to keep up with expectations, leading to reductions in investment and firm defaults.

The cycles produced by the present model correspond very closely to what is commonly understood as a ‘Minsky-cycle’ in the literature (cf. Nikolaidi and Stockhammer, 2017), being driven by investment expenditure and the related financing choices which, in turn, strongly depend on financing conditions and investors’ attitudes. Figure 6 shows that fluctuations in the average interest rate on loans are fairly modest, though this masks a larger dispersion and more pronounced volatility at the level of individual firms as shown below.
Figure 6: Average loan rate

Figure 7 gives an example of the dynamics of investment and the loan interest rate for one individual firm. It can be seen that investment of the individual firm is considerably more volatile than aggregate investment and includes fluctuations at much higher frequencies.

Figure 7: Investment (left) and loan rate (right) for an individual firm

Nevertheless, the strategic complementarities in investment decisions arising via the impact of investment on aggregate demand, GDP and profits lead to a sufficient correlation between the investment of individual firms to give rise to regular cycles at the aggregate level.

In order to gain a deeper understanding of the cycles, some simple experiments can be conducted. In particular, it appears reasonable to suspect that one or more of the non-
linearities incorporated into C-firms’ behavioural rules play a central role in producing the cyclical dynamics of the model. Equations 2, 6 and 10 represent three non-linearities which can easily be removed from the model by setting the parameters $\epsilon$, $\tau$ and $\beta_1$ & $\beta_2$ to zero respectively. In the case of $\epsilon$, this will produce constant mark-ups and hence eliminate price-competition between C-firms. Setting $\tau$ to zero makes the desired dividend payout ratio a constant. Setting $\beta_1$ and $\beta_2$ to zero eliminates the endogenisation of $\mu_1$ such that the borrowers’/investors’ risk parameter becomes a constant.

Setting $\epsilon$ to zero eliminates a great deal of firm heterogeneity from the model, making the aggregate cycles much more regular. The basic cyclical forces are still at work and aggregate cycles are in fact exacerbated relative to the simulation shown above. This suggests that a degree of price competition and the resulting firm heterogeneity in fact have a somewhat stabilising effect on the model. An elimination of the endogenous dividend payout ratio, on the other hand, fundamentally changes the dynamics produced. The aggregate cycles disappear completely with only minor and very irregular fluctuations remaining and the mean value of GDP being considerably higher than in the simulation shown above. The variable payout ratios and the associated financing choice of firms hence appear to play a central role in producing the cycle. By contrast, making $\mu_1$ a constant has a dampening effect on the fluctuations, suggesting that changes in investors’ risk-appetite have a reinforcing effect on the cycle.

Figure 8 shows the result of fitting theoretical distributions to the right tail of the size-distribution of C-firms in terms of sales. The plot is constructed by taking a snapshot of the distribution of sales at period 1000 across all 100 Monte Carlo repetitions.\textsuperscript{13}

\textsuperscript{13}It was found that the distribution is robust to the exact time at which the snapshot is taken. Because cycles do not coincide across Monte Carlo runs, the plot does not contain a ‘cyclical’ element.
It can be seen that the model gives rise to considerable heterogeneity among C-firms in terms of firm size, and both the log-normal and logistic distributions (for the former of which there is some empirical support, see e.g. Caiani et al. (2016)) appear to provide a decent fit. The size distribution hence seems to exhibit a heavy tail.

Figure 9 shows that the relationship between C-firms’ capital stocks and their profit rate significantly changes over the cycle. It plots the MC-average capital stock of each C-firm against its profit rate during a boom and a downturn, along with a locally fitted polynomial. It shows that while during booms, there is a strongly positive relationship between the capital stock and profit rate, during downturns this relationship breaks down to some extent. For firms with medium-to-high capital stocks there no longer appears to be a positive relationship between capital and the profit rate. Only for firms with very high capital stocks does the positive relationship between capital and profit rate remain intact.

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The boom period in each simulation is identified as the period one year (i.e. 4 quarters) before GDP reaches its global maximum while the downturn is the period 4 quarters before GDP reaches its global minimum.
The above analysis gives an idea of the added value of the agent-based approach taken in this model as compared to the aggregative approaches which have traditionally been employed in the Minskyan literature. Having presented the baseline simulation, I move on to conduct some experiments.

5 Simulation experiments

In the first simulation experiment, I alter the way in which C-firms form opinions about the prospective yield of capital goods. Instead of assuming that they simply use the current yield as a basis to calculate the demand-price of capital, I instead allow them to take either an optimistic or pessimistic attitude and adjust their investment strategy accordingly.\textsuperscript{15} In particular, for each firm a ‘sentiment’ variable is calculated which is given by

\[
\text{sentiment}^j = \frac{1}{1 + \exp(\eta(\hat{\pi}^j - \pi))}
\]

\textsuperscript{15}Another paper using a strategy switching approach in a Minsky model is Jump et al. (2017) in which firms switch between different strategies for the target debt to income ratio based on the mechanism proposed by Brock and Hommes (1997).
where \( \hat{\pi}_j \) is a weighted average of firm \( j \)'s profit rate and the average profit rate prevailing in the C-firm sector. Following this, a random number is drawn from a uniform distribution for each firm. If \( \text{sentiment}_j \) is greater than the random number drawn for \( j \), the firm in question takes an optimistic attitude (\( \text{attitude} = 1 \)), otherwise, it is a pessimist (\( \text{attitude} = 0 \)). Equation 8 is replaced by

\[
Q^j = \begin{cases} 
\left( \frac{c_j^j p^j - W^j}{r_d + \delta} \right)^{\text{opt}} & \text{if attitude} = 1 \\
\left( \frac{c_j^j p^j - W^j}{r_d + \delta} \right)^{\text{pes}} & \text{if attitude} = 0,
\end{cases}
\]

where \( \text{opt} > 1 \) and \( \text{pes} < 1 \). Figure 10 shows that while the cycles arising in the baseline model are preserved, investment becomes significantly more volatile. The standard deviation of capital investment increases by 19% relative to the baseline, with the explicit link from aggregate profitability to the sentiment of firms contained in equation 17 serving to strengthen the existing strategic complementarities in firms’ investment decisions, amplifying the cycle.

Figure 10: Capital investment with sentiment dynamics

This experiment highlights the central role played by investors’ expectations and beliefs in the two-price model. Building on the work of Keynes (1936), Minsky frequently emphasises the importance expectations and changing attitudes and beliefs for his theory of financial fragility, but this aspect has not always featured very prominently in formal
Minsky models, including those making explicit reference to the two-price model (papers in which expectations and beliefs do play an important role in various ways include Taylor and O’Connell (1985), Fazzari et al. (2008), Chiarella and Di Guilmi (2011) and Jump et al. (2017)).

In a second experiment designed to further investigate and illustrate the central role of beliefs and expectations, the model is modified such that C-firms are assigned a type at the beginning of each simulation, namely ‘baseline’ and ‘fundamentalist’. Baseline-type firms calculate the demand-price of capital exactly as in the baseline, using equation 8 to derive their belief about the prospective yield. Fundamentalist types, on the other hand, use the yield prevailing in the initial stationary state as their estimate, in the belief that this proxies some ‘fundamental’ value of the yield to which the actual one will revert. Apart from this, they behave exactly as do the baseline firms. In this experiment there is no switching behaviour; firms retain the type they are assigned at the beginning of the simulation for the entire duration of the run. If a fundamentalist firm goes bankrupt and is replaced by a new firm, this new firm is also assigned the fundamentalist type. The experiment is conducted by repeatedly running this modified version of the model with increasing shares of fundamentalist firms. The share of fundamentalist firms $s_f$ is increased from 0.1 to 1 in steps of size 0.1. Each parametrisation is run for 100 Monte-Carlo repetitions. At the beginning of each run, types are assigned randomly such that on average, the share of fundamentalist firms is equal to the value of $s_f$.

Table 5 shows the standard deviations and means of GDP generated by the modified version of the model for different values of $s_f$, relative to the baseline with $s_f = 0$. To make the standard deviations comparable, taking into account the differing mean values of GDP, they are calculated from the percentage-deviations of GDP from the respective post-transient simulation mean. It can be seen that the standard deviation of GDP decreases quite strongly with an increasing number of fundamentalist firms, reaching a value of 38% of the baseline when all firms behave as fundamentalists. Fluctuations in investment also become much less regular in the presence of fundamentalist firms as the incidence of herding behaviour present in the baseline is reduced. At the same time the
Table also shows the presence of fundamentalist firms has a strongly positive effect on the mean value of GDP relative to the baseline. In the baseline model, the mean value of GDP tends to settle below that prevailing in the initial stationary state whereas the presence of fundamentalist firms keeps mean GDP closer to its initial value. The presence of firms which believe the yield prevailing in the initial stationary state to be some type of ‘fundamental’ value is hence able to attract the realised time series closer to that initial stationary state, making fundamentalist firms’ beliefs at least in part self-fulfilling.

Table 5: Mean & standard deviation of GDP with fundamentalist firms relative to baseline

<table>
<thead>
<tr>
<th>$s_f$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rel. SD</td>
<td>0.80</td>
<td>0.63</td>
<td>0.54</td>
<td>0.48</td>
<td>0.43</td>
<td>0.41</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
<td>0.38</td>
</tr>
<tr>
<td>Rel. mean</td>
<td>1.06</td>
<td>1.11</td>
<td>1.15</td>
<td>1.18</td>
<td>1.23</td>
<td>1.27</td>
<td>1.31</td>
<td>1.35</td>
<td>1.39</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Overall this second experiment further emphasises the central role of firms’ beliefs or expectations in Minskyan investment cycles. Together with the first experiment, it shows that depending on how they are specified, beliefs about the prospective yield of capital goods can act as a type of coordination device which may produce herding behaviour that exacerbates economic fluctuations, or they may exert a stabilising effect on the economy. The next experiments explore possibilities to stabilise the baseline model through policy interventions.

Based on his views about the inherent instability of financialised capitalist economies, Minsky argued strongly in favour of public sector interventions, emphasising the necessity of ‘big government’ and ‘big bank’ to stabilise the economy (Minsky, 1986; Wray, 2016). ‘Big government’ is intended to stabilise the economy by maintaining aggregate demand and cash flows to the private sector through fiscal policy interventions which must be of a sufficient size relative to the overall economy. ‘Big bank’, i.e. a powerful central bank, must act as a lender of last resort to the financial sector, maintain the payments system and regulate financial markets. Importantly, Minsky argued that public policy must be designed to limit both the downward and the upward instability of the system, i.e. in
addition to intervening during recessionary periods it must also be designed to prevent
the emergence of unsustainable booms which eventually result in a period of crisis.

The strongly simplified structure of the model leaves only few possible policy levers. In
particular there is no straightforward way to examine the effects of a lender of last
resort and, by design, there cannot be a collapse in the payments system underlying the
model economy. A stylised version of fiscal policy can be implemented by assuming that
the banking sector, acting as an amalgamation of the financial sector, central bank and
government, makes transfer payments to the households according to

\[(19) \quad t = \phi_g(\hat{u} - u),\]

where \(u\) is aggregate industrial capacity utilisation with \(\hat{u}\) being a normal or target level.
Note that the transfer payment can be negative in which case it acts like a tax.\(^{16}\) Formally
this is exactly equivalent to the banking sector paying an additional dividend beyond its
current profit, with transfers exceeding current profits reducing the banks’ equity.\(^{17}\) As
far as central bank policy is concerned, the model structure allows for the implementation
of conventional (interest rate) monetary policy, which I introduce by endogenising the
parameter \(r_0\) which serves as the intercept in the equation for \(r_L:\)

\[(20) \quad r_0 = \max(0, \hat{r}_0 + \phi_b(u - \hat{u})),\]

where \(\hat{r}_0\) is set equal to the level of the exogenous \(r_0\) in the baseline model. Though Minsky
himself did not advocate the use of interest rate policy for macroeconomic stabilisation,
the structure of the two-price model in which the loan interest rate plays a key role does
suggest that interest rate policy should have an effect.

\(^{16}\)In the case of a positive transfer, this can be regarded as equivalent to a ‘money-financed’ government
budget deficit, or it might be interpreted as a fiscal operation of the monetary authority. From an
accounting perspective, the effect on household income and the households’ sectoral financial balance
is in either case equivalent to that of a government budget deficit financed through bond issuance (cf.
Fullwiler, 2013).

\(^{17}\)In some runs this may over the long term lead to banks’ equity becoming negative. As indicated
in section 3, this does by assumption not present a problem in the model as it is set up. Additionally
it should be noted that banks’ equity, whether positive or negative, enters into the wealth accounting of
households as owners of the bank.
Figure 11: Effects of monetary and fiscal policy

Figure 11 summarises the effects of the two policy regimes on the dynamics of GDP, showing that both fiscal and monetary policy lead to a considerable degree of stabilisation. Both monetary and fiscal policy are able to reduce the standard deviation of GDP by over 65% relative to the baseline and a stabilising effect of similar magnitude is achieved also when the baseline model is augmented by the sentiment dynamics outlined in the first simulation experiment above. Both monetary and fiscal policy also lead to a significant increase in average GDP relative to the baseline.

As argued by Minsky, fiscal policy stabilises aggregate demand and private sector cash flows, thereby indirectly exerting a stabilising influence on investment decisions not only through maintaining aggregate demand at the possible onset of downturns but also through curtailing booms which would eventually lead to increases in financial fragility. Monetary policy acts directly on the investment decision by increasing (decreasing) the supply price of capital at the onset of a boom (slump) and at the same time has an effect on the payment commitments of borrowers. The effectiveness of conventional monetary policy may appear somewhat unusual since interest rate policy as a stabilisation tool has played virtually no role in the Minskyan literature\textsuperscript{18} and, as pointed out above, Minsky

\textsuperscript{18}Indeed, it has variously been argued that interest rate policy may have a destabilising effect in a
himself focused on other aspects of central bank policy. Nevertheless, considering the specification of the two-price model in which the financing cost of investment plays a central role, and given the close link between central bank rates and bank lending rates (abstracting from possible issues of interest rate pass-through) it appears intuitive that interest rate policy would have a significant effect and could be stabilising if appropriately specified. An interesting extension of the framework presented here might hence involve the introduction of factors which could reduce the effectiveness of conventional monetary policy (as seems empirically plausible particularly in the case of major finance-driven cycles). A more detailed model could then also explore the types of central bank policy advocated by Minsky, including attempts at financial regulation.

Minskyan framework insofar as it has been posited as a possible factor bringing about the turning point at the peak of a cycle (cf. Nikolaidi and Stockhammer, 2017). This possibility should not be discounted in principle but in the present model it turns out that an appropriately specified interest rate rule can prevent such booms from emerging in the first place.
6 Conclusion

The purpose of this paper has been to provide a formalised dynamic version of Hyman Minsky’s two-price model of investment in a simple agent-based setting and to examine the emergent macroeconomic dynamics. Implementing the two-price model for a sector of agent-based consumption goods firms interacting with a set of strongly simplified stock-flow consistent aggregate sectors, the resulting model is calibrated empirically, showing that as far as the aggregate results of firms’ behaviour are concerned, it is capable of producing empirically plausible macroeconomic dynamics. The model gives rise to persistent investment-driven cycles at the aggregate level which emerge from the uncoordinated investment decisions of the agent-based firms. All this demonstrates that the microeconomic framework of Minsky’s two-price model can indeed serve as a suitable basis to depict dynamics in line with the Financial Instability Hypothesis. Beyond the use of the empirical calibration procedure, which is novel to the Minskyan literature, the paper represents an interesting contribution in that it provides a detailed formalisation of a central but relatively neglected aspect of Minsky’s work, whilst emphasising the added value provided by an agent-based approach in providing an additional layer of analysis to a literature which is still dominated by more aggregative modelling techniques.

The first simulation experiments showed that the incorporation of sentiment dynamics and strategy switching behaviour can exacerbate the macroeconomic fluctuations present in the baseline model, while the introduction of firms which behave as fundamentalists with regard to their prospective yield has a stabilising effect. This served to emphasise the importance of investors’ beliefs and expectations in the two-price model. The paper hence also adds to the small literature on Minsky models in which expectations formation plays a major role in producing the observed dynamics.

Finally, the model also allowed for the introduction of stylised monetary (interest rate) and fiscal policy interventions, and it was shown that both policies can lead to a very strong reduction in the volatility present in the baseline model, a finding which remains robust when the latter is augmented by sentiment dynamics. The finding regarding monetary
policy is surprising in the sense that interest rate policy has played a very limited role in
the Minskyan literature, but it was argued that the assumptions underlying the two-price
model do suggest that interest rate policy should have significant effects. This points to a
possible extension of the Minskyan literature which pays more attention to central bank
and especially interest rate policy and explores reasons why interest rate policy alone
may be an insufficient tool for macroeconomic management, linking to the rich existing
literature on this issue.
Bibliography


Appendix

Initialisation and parameter values

Table 6 shows the values of all parameters used in simulations and whether a given value is empirically calibrated (“emp”) imposed to produce an initial, consistent stationary state (“pre-SS”), implied by that stationary state (“SS-given”), or free.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_f$</td>
<td>free</td>
<td>Number of C-firms</td>
<td>50</td>
</tr>
<tr>
<td>$w$</td>
<td>pre-SS</td>
<td>Nominal wage</td>
<td>1</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>pre-SS</td>
<td>Normal mark-up</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>free</td>
<td>MPC out of income</td>
<td>0.85</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>SS-given</td>
<td>MPC out of wealth</td>
<td>0.03</td>
</tr>
<tr>
<td>$\theta$</td>
<td>pre-SS</td>
<td>Loan repayment rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\delta$</td>
<td>pre-SS</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$r_d$</td>
<td>pre-SS</td>
<td>Discount rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>SS-given</td>
<td>Borrowers’ risk parameter</td>
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</tr>
<tr>
<td>$\mu_2$</td>
<td>SS-given</td>
<td>Lenders’ risk parameter</td>
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<tr>
<td>$r_0$</td>
<td>pre-SS</td>
<td>Lending rate intercept</td>
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<tr>
<td>$\iota_1$</td>
<td>emp</td>
<td>Demand distribution price sensitivity</td>
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<tr>
<td>$\iota_2$</td>
<td>emp</td>
<td>Demand distribution size sensitivity</td>
<td>1.1</td>
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<tr>
<td>$\lambda_1$</td>
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<td>Persistence of demand distribution</td>
<td>0.75</td>
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<td>$\lambda_2$</td>
<td>emp</td>
<td>Persistence of demand distribution shock</td>
<td>0.38</td>
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<tr>
<td>$\sigma$</td>
<td>emp</td>
<td>Strength of demand distribution shock</td>
<td>2.6</td>
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<tr>
<td>$\kappa$</td>
<td>pre-SS</td>
<td>Full-capacity output to capital ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>$\chi$</td>
<td>free</td>
<td>Haircut applied to capital</td>
<td>0.75</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>emp</td>
<td>Borrowers’ risk adjustment parameter</td>
<td>0.4</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>SS-given</td>
<td>Borrowers’ risk adjustment parameter</td>
<td>0.8469388</td>
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<td>$\pi$</td>
<td>SS-given</td>
<td>Steady-state profit rate</td>
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<tr>
<td>$\gamma$</td>
<td>emp</td>
<td>Payout ratio intercept</td>
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<td>$\varepsilon$</td>
<td>emp</td>
<td>Price adjustment parameter</td>
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<td>Desired deposit to sales ratio</td>
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<tr>
<td>$\zeta_2$</td>
<td>free</td>
<td>Deposit adjustment parameter</td>
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<td>emp</td>
<td>Payout ratio adjustment parameter</td>
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<td>$\rho$</td>
<td>free</td>
<td>Sentiment index weighting parameter</td>
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<tr>
<td>$\eta$</td>
<td>free</td>
<td>Sentiment adjustment parameter</td>
<td>-20</td>
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<td>$opt$</td>
<td>free</td>
<td>Scaling parameter (optimistic firms)</td>
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<tr>
<td>$pecs$</td>
<td>free</td>
<td>Scaling parameter (pessimistic firms)</td>
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<td>$\phi_a$</td>
<td>free</td>
<td>Fiscal policy strength</td>
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<tr>
<td>$\phi_b$</td>
<td>free</td>
<td>Monetary policy strength</td>
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<tr>
<td>$\hat{u}$</td>
<td>free</td>
<td>Normal/desired capacity utilisation</td>
<td>0.8</td>
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Table 7 below shows the aggregate initial values which are needed to initialise the model for the simulations shown in the paper. Agent-based C-firms are initialised to be exactly
identical and each initial aggregate stock and flow is divided between firms according to their initial market share, \( \frac{1}{n_f} \).

Table 7: Initial values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>( Y )</td>
<td>SS-given</td>
<td>Nominal GDP</td>
<td>3375</td>
</tr>
<tr>
<td>( E_b )</td>
<td>pre-SS</td>
<td>Bank equity</td>
<td>300</td>
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<tr>
<td>( D_h )</td>
<td>SS-given</td>
<td>Household deposits</td>
<td>5400</td>
</tr>
<tr>
<td>( E_{fk} )</td>
<td>pre-SS</td>
<td>K-firm equity</td>
<td>0</td>
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<tr>
<td>( Y D )</td>
<td>SS-given</td>
<td>Disposable income</td>
<td>3000</td>
</tr>
<tr>
<td>( V_h )</td>
<td>SS-given</td>
<td>Household wealth</td>
<td>15000</td>
</tr>
<tr>
<td>( k )</td>
<td>pre-SS</td>
<td>Real Capital stock</td>
<td>10000</td>
</tr>
<tr>
<td>( K )</td>
<td>SS-given</td>
<td>Nominal capital stock</td>
<td>15000</td>
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<tr>
<td>( lev )</td>
<td>pre-SS</td>
<td>C-firms’ initial leverage</td>
<td>0.4</td>
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<tr>
<td>( L )</td>
<td>pre-SS</td>
<td>Stock of loans</td>
<td>6000</td>
</tr>
<tr>
<td>( r_L )</td>
<td>SS-given</td>
<td>Loan interest rate</td>
<td>0.04416667</td>
</tr>
<tr>
<td>( share )</td>
<td>pre-SS</td>
<td>C-firms’ market share</td>
<td>0.02</td>
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<tr>
<td>( D_{fc} )</td>
<td>pre-SS</td>
<td>C-firms’ deposits</td>
<td>300</td>
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<tr>
<td>( C )</td>
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<td>Nominal consumption</td>
<td>3000</td>
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<tr>
<td>( E_{fc} )</td>
<td>SS-given</td>
<td>C-firms’ equity</td>
<td>9300</td>
</tr>
<tr>
<td>( p_c )</td>
<td>SS-given</td>
<td>Consumption goods price level</td>
<td>1.5</td>
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