Non-performing loans and systemic risk in financial networks

Giulio Bottazzi
Alessandro De Sanctis
Fabio Vanni
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Abstract

In this paper we study the implications of non-performing loans (NPLs) for financial stability using a network-based approach. We start by combining loan-level data from DealScan and firm-level data from Orbis to reconstruct the empirical global financial network in the period 1991-2016 and identify a series of stylized facts. Based on these findings, we develop a model in which two types of agents, banks and firms, are linked in a network by their reciprocal claims and analyze how an increase in NPLs affects the stability of the system. We study the model analytically and with numerical simulations, deriving a synthetic measure of systemic risk and quantifying the threshold level of NPLs that triggers a systemic crisis. Our model shows that there exist a level of connectivity that maximizes the fragility of the financial system and that small changes in the initial NPLs shock can have very different consequences at the aggregate level.

Keywords: financial crisis; network theory; non-performing loans; resilience; systemic risk;

JEL codes: G21, C63, G01, D85

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†Scuola Superiore Sant’Anna (Italy). E-mail: giulio.bottazzi@sssup.it.

‡Paris School of Economics, Université Paris 1 Panthéon-Sorbonne (France). E-mail: alessandro.desanctis@psemail.eu. Corresponding author.

§OFCE SciencesPo (France). E-mail: fabio.vanni@sciencespo.fr.
1 Introduction

In this paper we study the implications of non-performing loans (NPLs) for financial stability, when different institutions, banks and firms, are linked to one another via direct bilateral exposures.

Our analysis starts from the observation of two phenomena. First, after the financial crisis of 2007-2008 the level of NPLs has significantly increased across the world. While in some countries, such United States, Japan or United Kingdom, this has been a transitory phenomenon, in others, like Italy and Spain, NPLs are still above pre-crisis levels (Figure 1) and represent a threat for financial stability, economic recovery and a factor of weakness in the event of future crisis.

Figure 1: Percentages of banks’ non-performing loans to total gross loans. Grey shaded areas correspond to U.S. recessions as defined by NBER. Data source: IMF, Global Financial Stability Report.

Second, over the last 30 years the level of connectivity and complexity of the financial system has grown enormously, with the result that today’s financial institutions are, directly or indirectly, much more connected than ever before.

While NPLs have always been recognized as a problem for financial stability, the view on connectivity has gone through different phases. Prior to the outbreak of the crisis, interconnectedness was seen as a way to diversify and reduce risk and hence not only was it positively gauged, but also encouraged by regulators. After 2007-2008 scholars and policy makers became aware of the drawbacks associated with an highly connected financial system and were forced to rethink how to promote financial stability. The new consensus is that interconnectedness is not an “unalloyed good”\(^1\) and that “too much” diversification may be suboptimal at the aggregate level, as it transforms risk from idiosyncratic into systemic.\(^2\)

\(^1\)“Interconnections among financial intermediaries are not an unalloyed good. Complex interactions [...] may serve to amplify existing market frictions, information asymmetries, or other externalities” (Yellen, 2013).

\(^2\)As pointed out by Stiglitz, the high number of interconnections between financial intermediaries “facilitated the breakdown” and became “part of the problem” (IMF conference “Interconnectedness: Building Bridges between Research and Policy”, May 2014, http://www.imf.org/external/pubs/ft/survey/so/2014/RES052314A.htm (accessed on 1 July 2015)). See also (Stiglitz, 2010) and (Battiston et al., 2012a).
In this context, it is natural to ask if the interplay between these two phenomena can pose further risks. For example, can an increase in the already high level of NPLs force one or more banks into defaults? Can such default(s) be propagated and amplified by the connections among financial institutions, ultimately leading to a systemic crisis? Is there a threshold level of NPLs beyond which a systemic crisis is likely to start? If yes, can we quantify it?

While the recent literature on financial contagion has explored extensively the role of interconnections in the propagation of a shock, it remained relatively vague on its origins. This missing link between models and reality limited the number of questions that they could answer and their usefulness from a policy point of view. Moreover, it prevented - at least to a certain extent - the possibility of linking models with data and of calibrating them.

Our paper tries to fill these gaps in the literature and to answer the questions posed above by developing a model that takes into account the interplay between NPLs and interconnectedness of the financial system. More in detail, we propose a model in which two types of agents, banks and firms, are linked in a network by their reciprocal claims and analyze how an exogenous increase in NPLs affects the stability of such system. We first focus on a simple version of the model with homogeneous fully-connected agents which we study analytically; then by means of Monte Carlo simulations, we investigate the more realistic case of fully heterogeneous agents.

Differently from the rest of the literature, we model the exogenous shock as an increase in the aggregate level of NPLs rather than the default of a given bank(s). The main advantage of using NPLs is that it allows to measure the intensity of the shock and to anchor it to an easily observable variable, making our modeling framework potentially useful for policy applications.

Our paper provides also some empirical evidence on the international financial network. We merge Orbis and DealScan data to reconstruct the set of lending and borrowing relationships between banks and between banks and firms from 1991 to 2016. This allows us to shed some light on the factors that drive the formation of financial networks and to identify several stylized facts which we use to guide the development of our model.

Our work proceeds as follows: section 2 reviews the literature and contextualize our paper; section 3 describes the data and illustrates the stylized facts; section 4 presents and discusses the model and section 5 concludes. In appendix A we discuss more in detail how we constructed the linking table between DealScan and Orbis. Finally, in appendix B we calibrate the model to Italy, Germany and United Kingdom and show how it can be used in stress-testing exercises.

2 Literature review

The one of financial networks and systemic risk is a relatively recent, but fast growing, field of research whose origins can be traced back to the work of Allen & Gale (2000) and Eisenberg & Noe (2001). Over the past few years this literature developed along two complementary directions: one part focused primarily on theoretical models of networks,

\footnote{As discussed later, the main impediment is still the scarcity of data.}
while another part devoted its attention to the empirical analysis of financial networks.  

Our paper lies somehow in between these two streams of literature, as it provides both a theoretical model and an empirical analysis of financial networks. From the theoretical point of view, the works of Elliott et al. (2014) and Montagna & Lux (2017) are the closest to ours. Elliott et al. (2014) develop a contagion model with homogeneous banks and a random network to study how default cascades depend on the network structure. Their main conclusions is that integration (greater dependence on counterparties) and diversification (more counterparties per organization) have different, nonmonotonic effects on the extent of cascades. Montagna & Lux (2017) studies default cascades in a scale-free network with heterogeneous banks. They use a fitness model to generate the network with the aim of reproducing some of the frequently documented features of inter-bank markets in terms of assortativity, degree distributions and nodes’ size distributions. Another seminal paper in this literature is Acemoglu et al. (2015). Here the authors study the role of the magnitude of the initial shocks on the extent of financial contagion and find that there is a form of phase transition: as long as the shocks are sufficiently small, a more connected financial network enhances financial stability; however, beyond a certain threshold, more interconnections serve as a mechanism for the propagation of shocks, leading to a more fragile financial system.

All these studies share the same mechanism behind the contagion dynamic, namely the amplification of the shock through direct bilateral exposures between nodes. There exist of course also other channels through which shocks can transmit and be amplified. For example “valuation effects”, which in a non-network context have been put forward by Kiyotaki & Moore (1997) and in a network context have been analyzed in Gai & Kapadia (2010). Another possible channel is the “lending channel”: here the default of a bank reduces lending supply, hence forcing some firms into default; this may in turn reduce the value of banks’ assets causing additional banks’ default and so on.

While these channels of contagion are certainly important, our study focuses on the direct channel only for three main reasons: first, to keep the model as simple as possible; second, because the indirect channels of shock transmission have, so to say, relatively easy dynamics and straightforward implications; third, because the increased connectivity of today’s financial system made the direct channel of contagion more and more important.

As for the empirical studies on financial networks, their number is significantly less than theoretical ones, mainly because of limited data availability. It is indeed very hard to reconstruct such networks and even supervisory authorities have typically incomplete knowledge. There are however several works that analyze empirically certain examples of financial networks. The networks of inter-bank payments are among the most studied since these payments travel through the central banks and are hence recorded. For example Soramäki et al. (2007) study the topology on inter-bank payment flows in the U.S. and find that the network is sparse, exhibits the “small world” phenomenon, is characterized by a scale-free degree distribution and is disassortative. To

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4 See Glasserman & Young (2016) for a detailed review of the literature.
5 Indeed a generalized fall in assets values will have surely a detrimental effects of balance sheets and hence increase monotonically the likelihood of failures.
6 Other relevant theoretical contributions which, for brevity of exposition, we omit to summarize here are: Gai et al. (2011); Battiston et al. (2012b); Georg (2013); Caccioli et al. (2014).
7 However, thanks to recent changes in the regulatory reporting, in many jurisdictions now supervision authorities have at least a partial knowledge of the linkages between financial institutions.
similar conclusions arrive De Masi et al. (2006) and Iori et al. (2008), which focus on the Italian overnight money market, and Linardi et al. (2019), which analyze the unsecured and the secured (repo) interbank lending network in Brazil. In addition to inter-bank studies, a subset of this empirical literature analyzes the financial network between banks and firms. An example is De Masi et al. (2011), which describes the Japanese credit network and find the existence of hubs (i.e. banks and firms with many connections), as well as a scale-free distribution of nodes’ degree. De Masi & Gallegati (2012) carries on a similar exercise for Italy and finds, again, a scale-free distribution of nodes’ degree and a positive correlation between nodes’ degree and size.

Compared to the above papers, the present study is - to our knowledge - the first to provide evidence on the international financial network. Moreover, our data allow us to study both the interbank and the bank-firm dimensions of financial networks, as well as the evolution through time. Finally, while most studies are based on data on short term lending activity, often overnight, our dataset contains both short and long term maturities (although in this paper we do not investigate in detail this dimension).

3 Empirical evidence

This section describes how we constructed the dataset used in our study and presents several stylized facts that characterize the network of borrowing-lending relationships among banks and firms at international level.

3.1 Data

The two source of data employed are Orbis and DealScan. From Orbis we get balance sheet information for companies across the globe from 1991 to 2016, while from DealScan we extract detailed information about bank loans from 1980 to 2018.

Orbis and DealScan do not contain a common identifier that can be used to merge the two datasets. Therefore we developed a text matching algorithm to link companies’ names and then we visually double check the quality of the matches.

The problem of merging DealScan with other firm level databases has been encountered also by other researchers and it has been overcome by matching the company names similarly to what we do. In particular, the two most used linking tables are from Chava & Roberts (2008) and Schwert (2018). Chava & Roberts (2008) focuses on the borrowers in DealScan and matched their names with Compustat names using a fuzzy matching algorithm. They were able to match 6716 unique firms, successively extended to 29502 in later versions of the table. Schwert (2018) focuses on the lenders associated to the loans matched in Chava & Roberts (2008). From this set of loans the author manually matched...
the most active lenders in DealScan with their respective bank holding companies in Compustat and BankScope. Overall the linking table in Schwert (2018) provides a match with Compustat for 752 lenders and with BankScope for 611 lenders.

Similarly to Chava & Roberts (2008) we use a fuzzy match algorithm to match name, but we focus both lenders and borrowers in DealScan. Moreover we do not match them with Compustat, but with companies in Orbis. The use of Orbis allows us also to extend the matching to non-U.S. companies and hence investigate the international dimension of DealScan data.

The resulting linking table\(^9\) between DealScan and Orbis contains 39515 matched companies (both lenders and borrowers) and it covers about 80% of the total value of transactions recorded in our DealScan extraction. More details about the creation of the linking table are available in Appendix A.

Using the linking table we are able to merge DealScan and Orbis and obtain a dyadic panel data structure, with information on each loan and the associated lender and borrower. Lenders and borrowers in DealScan can be linked by multiple lines of credits (also called facilities or loans). In order to make the data comparable with our theoretical model, we aggregate the loans at year-lender-borrower level, so as to get the bilateral exposures (or gross loan issuance) between nodes pair at each point in time. The final dataset span from 1991 to 2016 and includes about 1.8 millions of observations.\(^{10}\)

A limit of DealScan is that it contains mostly large syndicated loans. Single-bank and smaller loans are also present in the dataset, but the sample is nevertheless by construction biased towards big banks and big firms. While a more representative sample would obviously be desirable, for our purpose this characteristic does not constitute a major problem, as there is still sufficient heterogeneity in the data to derive the stylized facts of interest.

To have a closer matching with our theoretical model, in what follows we divide our dataset in loans between banks and firms and between two banks\(^{11}\). A potential concern here is that our data are not well suited to investigate inter-bank lending. Indeed DealScan has mostly observations on bank-firm lending. However we find that a significant number of loans occur between two banks, therefore, DealScan still provides useful hints about lending activity between banks. Interestingly, inter-bank lending occurs mostly across boarders, typically from the United States to Asian countries.

Figure 2 shows some basic statistics about the DealScan data. In general all our variables appear to be correlated with the level of economic activity at world level and in particular in the U.S. and Asia, as shown by the trough around the U.S. recessions and the Asian crisis of 1997. More in detail, panel (a) shows the evolution of the bank-firm

\(^9\)The linking table can be useful for other researchers that want to combine DealScan and Orbis data, therefore we made it freely available for download on our websites.

\(^{10}\)It is still possible to have companies in a given year (either lender or borrower) for which we do not have information on a particular variable (e.g. the total assets), therefore in the plots that follow the number of observation can be lower.

\(^{11}\)DealScan provides informations on the type of company involved in a transaction, such as whether it is a bank or not. However this information is missing for some companies. In this case, we assume that all the borrowers are firms, except when they have the word ‘bank’ in the name. This is consistent with the evidence we gain when we limit ourselves to the observations for which we have information of the node’s type, namely that the vast majority of lending activity goes from banks to firms, a smaller part goes from banks to banks, a handful of observation show firms lending to banks and one observation shows a firm lending to another firm.
lending activity. The number of active nodes - borrowers (in red) and lenders (in blue) - increased steadily until 1996. This is likely to be due both to the widening coverage of the dataset, both to the increasing financialization and globalization of financial markets. With the Asian crisis of 1997-1998 the number of nodes drastically decreased.\textsuperscript{12} Since then, the number of lenders remained essentially stable, while borrowers kept rising (with the exception of recession periods). Moreover, the grey line shows the so called average degree for the lenders\textsuperscript{13}, that is the average number of counter-parties of a lender in a given year. As it can be seen the average degree also increased substantially, from about 15 in 1991 to about 35 in the years just before the financial crisis, and still now it is close to its maximum. Panel (c) shows that the number of loans (links) and the volume transacted are also characterized an increasing trend, again with the exception of recessions. Moreover, the amount and the number of loans appear to be highly correlated, meaning that the average value of loans remained rather constant through time.

Panel (b) shows the evolution of the number of the inter-bank lending activity. After a constant increase until 1996, the number of active nodes decreased suddenly after the Asian crisis and again after the 2007 crisis. Interestingly, contrary to bank-firm lending, the reduction in the number of active nodes after a crisis does not appear to be temporary, but rather permanent\textsuperscript{14}. In terms of number of loans and volume transacted, panel (d) shows that compared to the inter-bank the evolution here is more lumpy and not marked by a clear trend, mostly because of the lower volume of transactions and nodes active. However these two measures still appear to be correlated with the general economic conditions and between themselves, meaning that on average the amount of a single loan did not vary much through time.

All in all, the evidence from Figure 2 confirms the idea that over the last 30 years globalization and risk diversification increased the connectivity and complexity of the financial system, reducing world’s size “from XXL to Small” (Friedman, 2006). Financial institutions are now directly or indirectly much more connected than ever before and a shock can therefore spread more easily and further than in the past.

\textsuperscript{12}Indeed, while about 40% of the transactions involve U.S. actors, Asian lenders and borrowers constitute the second largest group of actors in the dataset.

\textsuperscript{13}We show only the lender average degree for comparability with our theoretical model. A similar pattern can be observed for the borrower average degree.

\textsuperscript{14}The reasons can be multiple and an explanation of this phenomenon is beyond the scope of this work, but one possible cause is the consolidation process that took place in the banking industry at global level in the last decades and which reduced the number of active players.
Figure 2: Evolution of the number of active nodes, the number of links formed and the total amount transacted (as of end of year). Grey shaded areas correspond to U.S. recessions as defined by NBER.
3.2 Stylized facts

In this section, we document a number of empirical stylized facts which we use to guide the development of the model in Section 4. In what follows we take 2011 as reference year, as it is the one with the highest data quality. However similar patterns can be found in the other years too.

The evidence presented highlights three main ideas: (i) nodes’ size is an important determinant of the probability of forming links; (ii) the larger is a node the more is going to be connected; (ii) the amount of the loans is proportional to the size of the involved parties.

Fact 1. *The degree distributions are very skewed and follow approximately a power law.*

In other words there are very few highly connected nodes (hubs) and many weakly connected ones. This gives rise to a scale-free network structure, which has been found also by other studies on different financial network datasets. Figure 3 shows the empirical cumulative degree distributions for the borrower (in-degree) and lender (out-degree) in both the bank-firm and the inter-bank networks for various years\(^\text{15}\). The degree in a given layer of the network (bank-firm or inter-bank) is calculated taking into considerations only the connections in that network. Therefore if for example a bank lends to both firms and banks, it will have an higher total degree, equal to the sum of the bank-firm degree and the inter-bank degree. As it can be seen, the distributions are approximately a straight line, indicating the presence of a power law. It is interesting to note that, coherently with the evidence in Figure 2, the degree distributions changed through time. In particular, panel (a) shows how the distribution of the borrowers progressively moved to the left, while still spanning on the same range, meaning that it became more and more skewed. In panel (b) the degree distribution of lenders moved to the right and slightly increased the support, confirming an expansion of international lending activity. Panel (c) shows how the distribution for borrowers remained essentially unchanged, notwithstanding the reduction in the number of borrowers seen in panel (c) of Figure 2, which may have been counteracted by an increase in the lending activity (panel (d) Figure 2). Finally, panel (d) shows that the degree distribution of lenders first moved to right and then to left. Coupled with the evidence of Figure 2, we can interpret the right and left shift as a signs of increasing and decreasing lending activity.

\(^{15}\)The same pattern holds in all years from 1991 to 2016.
Figure 3: The x-axis gives the degree of the node, presented on a log scale. The y-axis, also in log scale, gives the probability of finding a node with a degree larger than or equal to x, that is the empirical counter-cumulative distribution (CCDF).
**Fact 2.** Larger nodes have higher degree.

There is a positive correlation between the degree, namely the number of different counterparties, and the size of a node. Figure 4 shows that the size, measured by total assets, is associated with an higher number of links. This is true in the bank-firm and in the inter-bank network for both lenders and borrowers.

![Figure 4](image1.png)  
(a) Bank-firm layer  
(b) Bank-firm layer  
(c) Inter-bank layer  
(d) Inter-bank layer

Figure 4: Linear regression of firm-level log degree (y-axis) on log total assets (x- axis) in 2011. Gray dotted lines denote the 95 percent confidence bands.

**Fact 3.** The size of the node and the amount of the loan are positively correlated.

This means that, given the size of the counter-party, larger nodes tend to receive (make) bigger loans, as shown in Figure 5. This is a very natural condition and a possible explanation is that larger borrower need more credit and are also able to obtain it, since they are considered more trustworthy than smaller ones. Similarly smaller borrowers need less credit or are financially constrained since they are perceived as more risky. On the lender side, larger banks have more financing capabilities and are able to grant larger loans, while smaller banks can extend only smaller loans.

![Figure 5](image2.png)
Figure 5: Linear regression of firm-level log average loan amount (y-axis) on log total assets (x-axis) in 2011. Gray dotted lines denote the 95 percent confidence bands.
Fact 4. The international financial network is characterized by a disassortative behaviour. Figure 6 shows the correlation between the degree of nodes and the average degree of their immediate neighbours\(^{16}\). As it can be seen, in all panels the fitted line is downward sloped, meaning that highly connected nodes tend to connect with weakly connected ones, giving rise to a disassortative network. This is coherent with the findings in the literature and an additional symptom of a scale-free network structure.

![Graphs showing degree assortativity in 2011.](a) Bank-firm layer (b) Bank-firm layer (c) Inter-bank layer (d) Inter-bank layer

Figure 6: Degree assortativity in 2011. On the x-axis the possible values of degree, on the y-axis the corresponding average degree of the connected nodes. The fitted regression line is denoted by the solid red line.

4 Model

We now outline a model of financial network which is inspired by the finding of the previous section. However, while in section 3 we dealt with international data, for the sake of simplicity in the theoretical model we abstract from this dimension, since it would not add much to the transmission mechanism in which we are interested. Extending the model in this direction and studying the international transmission of financial shocks, remains anyway a natural and interesting possibility that we intend to investigate in a future work.

\(^{16}\)Also known in the network literature as the average nearest neighbours degree.
4.1 Set up

Consider an economy composed by $N$ banks and $M$ firms, linked one another by their reciprocal credits and debits. Assume that banks and firms are represented as nodes in a network and that for any node an incoming link is a credit and an outgoing link is a debit. We assume that banks can lend and borrow from other banks, but can only lend to firms; moreover we assume that firms cannot borrow from each other, but only from banks. These assumptions imply that banks can have both incoming and outgoing links with other banks, but only incoming links form firms. On the other hand, firms can only have outgoing links toward banks and no links with other firms. The two sets of nodes form a bipartite network organized in two interconnected layers, where one comprises banks and the other firms, as shown in Figure 7.

![Figure 7: Stylized example of a network with two layers: in green the inter-bank layer, where banks lend and borrow from each other; in gray the bank-firm layer, where banks act only as lenders to firms.](image)

Green nodes and gray nodes in Figure 7 represent respectively banks and firms. Nodes’ size and links’ weight are ignored for simplicity. Note that two banks can be linked with each other in both directions, as in the case of $B_1$ and $B_2$, since we do not net inter-bank positions. In this example all banks and firms have at least a link, however as in our model the formation of links is probabilistic, it is possible to have completely disconnected banks or firms. The only constraint on the structure of the network is that self-loops, namely links starting and ending in the same node, are not allowed.

4.2 Balance sheet structure

Following Nier et al. (2007), we represent each bank via a simplified balance sheet structure as the one depicted in Table 1. The total assets of bank $i$ are composed by the inter-bank assets $A_i^{IB}$, that is the total money lent to other banks, and by the external assets, $A_i^F$, that is the total money lent to firms, so that $A_i^{TOT} = A_i^{IB} + A_i^F$. At the same time, the banks liabilities are composed by money borrowed by other banks, $L_i^{IB}$ and deposit, $D_i$. Due to the double-entry bookkeeping system, the total assets are equal to the total liabilities so that the capital (equity) of the bank, $K_i$, is defined as

$$K_i = A_i^{IB} + A_i^F - L_i^{IB} - D_i.$$ 

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17 This assumption rather natural and finds confirmation in our loan data.
18 This is consistent both with the literature and with bankruptcy laws in many countries.
Table 1: Balance sheet structure. The arrows on the top of the figure indicate the direction of links: incoming links represent an asset, outgoing links a liability. The shock affects the portion of the assets held against firms. To cause the default of a node, the initial shock must be higher than the net worth owned by that node. Elaboration of the authors adapted from Haldane & May (2011).

A generic firm $j$ is uniquely described by its total assets $F_j$. While banks in our model possess heterogeneous levels of inter-bank assets, we make the simplifying assumption that they have the same portfolio composition and leverage ratio. More precisely, we assume that a fraction $\theta \in (0,1)$ of assets comes from inter-bank lending and a fraction $1 - \theta$ from lending to firms, while, at the same time, the leverage, i.e. the asset equity ratio, is fixed to be $1/\eta$. Formally one has

$$A_i^{IB} = \theta A_i^{TOT}, \quad A_i^F = (1 - \theta)A_i^{TOT} \quad \text{and} \quad K_i = \eta A_i^{TOT}. \quad (1)$$

With the assumptions above, the bank deposits are computed as a difference between $A_i^{TOT}$ and $L_i^{IB} + K_i$. Bank deposits can be both positive or negative. In the latter case, they should be considered assets owned by the bank. In any case, they represent positive or negative bank exposure to the risk-free part of the economy.

4.3 Network formation

Each bank $i = 1, \ldots, N$ and each firm $j = 1, \ldots, M$ is initialized with a given level of total assets, respectively labeled $A_i^{TOT}$ and $F_j$. We take node’s total assets as proxies for the size and we use them as fitness parameters in the linking function with which we generate the network. Following the evidence provided in Fact 2, which shows that larger nodes tend to have more links, we assume that the probability $p_{i,j}^{IB}$ of generating a link between

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19 We do not try to precisely model the balance sheet structure of firms since our focus is on the consequences of shocks for the financial system.

20 In accordance with the current literature we label this variable “deposits”, however it must be noted that this is just a convention and this variable has no direct relation with real banks’ deposits.
bank $i$ and bank $j$ is proportional to the size of the nodes

$$p_{i,j}^{IB} = \left( \frac{A_{i}^{TOT}}{A_{max}^{TOT}} \right)^{\alpha} \left( \frac{A_{j}^{TOT}}{A_{max}^{TOT}} \right)^{\beta} \quad \text{with} \quad \alpha, \beta > 0 \quad (2)$$

Similarly the probability $p_{i,j}^{F}$ of generating a link between bank $i$ and firm $j$ is proportional to the total assets of the two nodes.

$$p_{i,j}^{F} = \left( \frac{A_{i}^{TOT}}{A_{max}^{TOT}} \right)^{\phi} \left( \frac{F_{j}}{F_{max}} \right)^{\chi} \quad \text{with} \quad \phi, \chi > 0 \quad (3)$$

Where the values of the exponents in 2 and 3 reflect the relative importance of the two nodes in forming a link, that is a bigger value of the exponent of the lender node means that the lender plays a bigger role in the "decision" of forming a link.

We randomly generate the network starting with $N$ isolated nodes and create links according to the probabilities in (2) and (3). The obtained directed network can be described by variables $l_{i,j}^{IB}$ and $l_{i,k}^{F}$ with $i,j = 1 \ldots N$ and $k = 1 \ldots M$: the first takes value one if a link exists from bank $j$ to bank $i$ and zero otherwise, the second takes value one if a link exists from firm $k$ to bank $i$ and zero otherwise. By choosing the number of active links, we can exogenously fix the final average degree of the inter-bank network $AD_{B,B} = \sum_{i=1}^{N} \sum_{j=1}^{N} l_{i,j}^{IB}/N$, defined as the number of inter-bank links over the number of banks, and the average degree of the inter-bank $AD_{B,F} = \sum_{i=1}^{N} \sum_{j=1}^{M} l_{i,j}^{F}/N$, defined as the number of links between banks and firms over the number of banks.

The model described is able to reproduce the disassortative behavior in nodes degree documented in Fact 4\textsuperscript{21}. Indeed, in a factorized linking probability model, the disassortative behavior emerges naturally, as described by Catanzaro \textit{et al.} (2005) and Caldarelli (2007)\textsuperscript{22}.

After having generated the network, we assign a weight to each link, which represents the amount of money borrowed by a node (bank or firm) and lent by another node (bank). Following the evidence provided in Fact 3, which shows that the loan amount is proportional to the size of the two nodes, we distribute the amount of inter-bank assets and of external assets of each bank proportionally to the size of the borrower. In particular, for inter-bank links, the weight depends on the amount of inter-bank assets of the creditor ($A_{i}^{IB}$) and on the size of the debtor ($A_{j}^{TOT}$), as well as on the number of incoming links of the creditor. Formally, the amount of money bank $j$ owns to bank $i$ is

$$w_{i,j}^{IB} = l_{i,j}^{IB} A_{i}^{IB} A_{j}^{TOT} \sum_{k=1}^{N} l_{i,k}^{IB} A_{k}^{TOT}. \quad (4)$$

Similarly, the weights of bank-firm links depend on the external assets of each bank $A_{i}^{F}$, on the level of total assets of firms $F_{j}$ and on the number of incoming links of the bank.

\textsuperscript{21}On this, see also the contributions mentioned in Section 2.

\textsuperscript{22}More in detail, Catanzaro \textit{et al.} (2005) shows that disassortativity emerges due to the finite size of the network and it is linked to the so-called structural cutoff. According to Caldarelli (2007), in the asymptotic case of an infinite number of nodes, for this model the average degree of nodes linked to a specific node is expected to be independent upon the degree of the node itself.
Formally, the amount of money firm $j$ owns to bank $i$ is

$$w_{i,j}^F = \frac{F_j}{\sum_{k=1}^{M} \frac{F_k}{t_{i,k}^F}} .$$

(5)

The weight above preserve the previously determined amount of inter-bank and external assets, $\sum_{j=1}^{N} w_{i,j}^\text{IB} = A_i^\text{IB}$ and $\sum_{j=1}^{M} w_{i,j}^F = A_i^F$ and, at the same time, guarantees the very natural condition that the amount involved in a financial transaction increases proportionally with the size of the involved parties.

Following this procedure we obtain a network that is bipartite, directed, and weighted and whose structure depends on the distribution of banks’ and firms’ size.

In our procedure, inter-bank liabilities of each bank are endogenously determined and we are able to compute all the elements of banks’ balance sheet in Table 1. In particular, the inter-bank liabilities of bank $i$ rest defined as

$$L_i^\text{IB} = A_i^\text{IB} \sum_{j=1}^{N} \frac{l_{j,i}A_j^\text{IB}}{\sum_{k=1}^{N} l_{j,k}A_k^\text{IB}}$$

and the deposit

$$D_i = A_i^\text{IB} \left( \frac{1 - \eta}{\theta} - \sum_{j=1}^{N} \frac{l_{j,i}A_j^\text{IB}}{\sum_{k=1}^{N} l_{j,k}A_k^\text{IB}} \right).$$

The latter expression can be interpreted as deposit if positive, or risk-free assets, for instance household mortgages, if negative. The total exposition of the banking system to the riskless part of the economy is however positive and proportional to the overall amount of inter-bank assets

$$\sum_{i=1}^{N} D_i = \frac{1 - \eta - \theta}{\theta} \sum_{i=1}^{N} A_i^\text{IB} .$$

### 4.4 Bankruptcy cascades and amplification mechanism

After having initialized the model, we perturb the system at time $t = 1$ with an exogenous shock consisting in an increase in the level of NPLs due to firms default. The idea is that when a firm defaults it is no longer able to repay its debt, so the banks who granted it a loan mark it as non-performing and write-down the corresponding book value. Banks exposed toward defaulted firms incur in a loss which erodes their capital, potentially forcing them to default. In practice some of the credits provided by banks to firms are transformed in bad loans and their value is set to zero. We do so by selecting firms at random and assuming that they become unable to meet their obligations until we reach the desired amount of NPLs. More in detail, given an amount $\delta$ of NPLs, we select a firm at random; again at random we go through its outgoing link one by one; we set the value of the selected link equal to zero and we repeat until an amount of debt equal to $\delta$ is cancelled. If the total debt of the firm is greater than $\delta$, the last link considered is simply reduced by the amount necessary to reach $\delta$. If instead the total debt of the firm

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23For example, a power law distribution generates a scale-free type of network, as in the “Heterogeneous” case discussed in Section 4.6.
is lower than \( \delta \), the procedure continues with another randomly selected firm, until the debts of all the link cancelled is equal to \( \delta \). Notice that this way of selecting firms at random makes the distribution of firms’ size relevant, as we select with higher probability the most frequent type of firm\(^{24}\).

After this initial process is over, one or more banks have their external assets reduced below the initial level, \( A_{F,1}^i < A_i^F \). If the amount of capital of bank \( i \) is reduced such that it is unable the to meet the solvency condition
\[
K_i = A_i^{\text{IB}} + A_i^F - L_i^{\text{IB}} - D_i > \rho A_i^{\text{TOT}} \quad \text{with} \quad \rho \geq 0 ,
\]
the bank becomes insolvent and it is set to default. Essentially we impose a minimum capital requirement on banks expressed as a fraction \( \rho \) of their total assets. All defaulted banks are assumed to default on all their liabilities and for all the amount.\(^{25}\)

At this point a new round \( t = 2 \) starts. Assets corresponding to loans to previously defaulted banks are set to zero and the inter-bank assets \( A_i^{\text{IB}} \) of the creditor banks are accordingly reduced. The solvency condition (6) is checked again and the banks that are now unable to fulfill it are set to default. This process continues to iterate until no further bank failures occur. In this way the initial firm-level shock transmits at inter-bank level where failed banks are assumed to default on all of their inter-bank liabilities, eventually pushing neighbour banks into default. The initial shock can be either absorbed or amplified, eventually triggering a cascade of defaults able to cause a systemic crisis within the financial network. Notice that liabilities are not adjusted following banks default. This assumption reflects the consideration that the process of deleveraging is typically much longer that the development of the default cascade and generally cannot help in preventing the bankruptcy.

It is useful to analyze the role played by the portfolio composition parameter \( \theta \), and the leverage parameter \( \eta \), in starting and sustaining a default cascade. Consider a specific bank \( i \) and let \( \delta \) be the fraction of external assets that have become non-performing after the initial shock, so that \( A_{F,1}^i = (1 - \delta)A_i^F \). Since initial total assets are equal to total liabilities, it is \( L_i^{\text{IB}} + D_i = A_i^{\text{TOT}} - K_i \) and substituting (1) in (6) after little algebra the solvency condition simply becomes
\[
\delta < \delta^* \quad \text{with} \quad \delta^* = (\eta - \rho)/(1 - \theta) .
\]
The critical threshold \( \delta^* \) is the minimum level of initial shock \( \delta \) on external assets that sets a bank \( i \) into default. As expected, this is a decreasing function of the leverage \( 1/\eta \) and an increasing function of the share of inter-bank assets \( \theta \). In subsequent rounds, assume that bank \( i \) has lost a fraction \( \delta^{\text{IB}} \) of its inter-bank assets. Using the same procedure, one can rewrite the solvency condition as
\[
\frac{\theta}{1 - \theta} \delta^{\text{IB}} + \delta < \delta^*. \quad \text{(8)}
\]
\(^{24}\)For example if firms are distributed according to a power law, then we will observe many defaults among small firms and few ones among big firms.

\(^{25}\)As pointed out in Gai & Kapadia (2010) “this assumption is likely to be realistic in the middle of a crisis: in the immediate aftermath of a default, the recovery rate and the timing of recovery will be highly uncertain and banks’ funders are likely to assume the worst-case scenario”. Anyway it would be possible to relax this assumption and allow for a partial recovery, so that when a linked bank defaults, the creditors do not lose all their asset, but get some fraction of it, for example a share of the remaining assets proportional to the weight of creditors’ asset over all other liabilities of the defaulted bank.
A few comments are in order. First, notice that the parameter \( \rho \) enters into the definition of \( \delta^* \) only as a modifier of the leverage \( \eta \), thus without loss of generality we can assume \( \rho = 0 \). This corresponds to the case in which a bank is considered insolvent when its equity becomes zero or negative. Second, the role of the leverage is obvious, as it decreases the resilience of the network: higher leverage levels (lower \( \eta \)) makes banks more exposed to the failure of both firms and other banks. Conversely, the portfolio composition has two opposite effects on the default cascade. On the one hand, a higher value of \( \theta \) increases \( \delta^* \) and shields banks from possible default in round 1, when firms loans becomes non-performing. On the other, it increases the exposition of banks to the failure of other banks in the successive rounds. There exists therefore a level \( \theta^* \) that maximizes the number of defaults and the fragility of the system\(^{26}\). However, since \( \delta^{IB} \) is the cumulative shock that a single bank receives round after round from the inter-bank network (and hence is a function to the network structure and \( \theta \) itself) the value \( \theta^* \) can be found only with computationally intensive simulations, which we were not able to run with our machines.

4.5 Analytical discussion of the model in the homogeneous fully-connected case

To understand how the model works it is useful to study the case of fully connected networks and homogeneous nodes, that is \( A^i_{IB} = A^B \) and \( A^j_F = A^F \) for \( i = 1 \ldots N \) and \( F_j = F \) for \( j = 1, \ldots, M \). In this case each firm owns an amount \( A^F/M \) to each bank and the aggregate external debt of banks is \( NA^F \). Let \( \lfloor x \rfloor \) denotes the integer part of \( x \) and \( \{x\} = x - \lfloor x \rfloor \) its fractional part. One has the following

**Proposition 4.1.** Let \( \delta \) be the fraction of initially defaulting loans, then

- if \( \lfloor \delta M \rfloor \geq \delta^* M \) all banks initially default.
- if \( \delta^* M - 1 \leq \lfloor \delta M \rfloor < \delta^* M \) exactly \( \lfloor N \{\delta M\} \rfloor \) banks initially default;
- if \( \delta^* M - 1 - \{N \{\delta M\}\} \leq \lfloor \delta M \rfloor < \delta^* M - 1 \) a single bank initially default;
- if \( \lfloor \delta M \rfloor < \delta^* M - 1 - \{N \{\delta M\}\} \), no banks initially default;

**Proof.** The number of completely defaulting firms is \( \lfloor \delta M \rfloor \). Their NPLs generate a loss equal to \( \lfloor \delta M \rfloor A^F/M \) for each bank. According to (7), if \( \lfloor \delta M \rfloor /M \geq \delta^* \) then all banks initially default and the first point is proved.

The following points can be proved analogously by observing that after the complete default of the \( \lfloor \delta M \rfloor \) firms, a faction of NPLs equal to \( NA^F\{\delta M\}/M \) still has to default. This generate a further loss of \( A^F/M \) for \( \lfloor N \{\delta M\} \rfloor \) banks.

Finally, the last loan is affected by a partial default equal to \( A^F\{N \{\delta M\}\} \).

If some banks survive the initial NPLs shock, since they are all identical, for symmetry argument, only two possibilities arise: or they all default in the first round of the bankruptcy cascade or they never default. Specifically one has the following

\(^{26}\)As discussed in Section 5 \( \theta \) is a key policy parameter and it has an interesting, tightly relation with the Glass-Steagall Act.
Proposition 4.2. If 
\[ [\delta M] \geq \delta^* M - \frac{\theta}{1 - \theta} \frac{M}{N - 1} [N\{\delta M]\] 
then all banks surviving the initial NPLs shock will default. Otherwise, no banks further default after the first NPLs shock.

Proof. All banks not defaulting for the initial shocks absorb a further loss due to the failing banks equal to \( A^{IB} [N\{\delta M]\]/(N - 1) \), that is
\[ \delta^{IB} = \frac{[N\{\delta M]\]}{N - 1} \]
using (8) the statement immediately follows.

The joint effect of Propositions 4.1 and 4.2 is that when \( N, M \to \infty \), the fully connected model has an abrupt phase transition: for \( \delta \geq \delta^* \) all banks default, while for \( \delta < \delta^* \) none does. Introducing heterogeneity will change the picture, however, as discussed through numerical examples in the next section.

4.6 Monte Carlo simulations

We start the numerical investigation of the model considering two cases: one with homogeneous and one with heterogeneous banks and firms. The corresponding values of the parameters are reported in column “Homogeneous” and “Heterogeneous” of Table 2. They are derived from the ones usually assumed in the literature (Gai & Kapadia (2010), Montagna & Lux (2017), Upper (2007), Nier et al. (2007)).

As seen in the previous section, in the homogeneous fully-connected case, if \( \delta < \delta^* - \frac{2}{M} \) no bank default may occur, while if \( \delta > \delta^* \) all banks default. Given the value of the parameters it is \( \delta^* = 0.1 \), so that the whole transition from total safety to financial mayhem happens when \( \delta \) increases from 0.096 to 0.1. In order to investigate the role of network topology, we consider instead the scenario in which all banks and all firms still have the same size, that without loss of generality we can set equal to 1, but random networks are generated with different average degree of the bank-bank and inter-banks.

The average degree of the bank-bank and inter-bank is the key parameter and it is a proxy for both the level of interconnectedness of a system. We consider different combinations of \( AD_{B,B} \) and \( AD_{B,F} \), exploring all the range of possibilities from a collection of isolated nodes to a fully connected network. For every pair of values we generate 200 realizations of the network. Then, for each realization of the network, we shock the system by increasing the level of NPLs as described in Section 4.

Since we are interested in the risk of a systemic crisis we want to exclude small chain of defaults. For this reason, following Gai & Kapadia (2010), we define a systemic crisis as the occurrence of the default of more than 5% of banks in the network. Given this definition, we compute the frequency of a systemic crisis \( (F) \) as the number of times in which more than 5% of banks default over the 200 drawings and the extent of a systemic crisis \( (D) \) as the fraction of defaulted banks conditional on contagion over the 5% threshold breaking out, which is therefore a measure of the magnitude of the systemic crisis. These two quantities allow to define a synthetic statistics for measuring systemic risk \( R = F \times D \), computed as the product between the frequency of contagion, \( F \), and the extent of contagion, \( D \).
Table 2: Values of the parameters used in the Monte Carlo simulations. \( N \) and \( M \) are respectively the number of banks and firms in the network; \( \eta \) is the fraction of capital with respect to total asset; \( \theta \) is the fraction of inter-bank assets with respect to total assets. \( \delta^* \) is the solvency critical threshold assuming the corresponding values of \( \eta \) and \( \theta \). As for the exponents of the linking functions in equations 2 and 3, we follow the literature (see Montagna & Lux (2013)) and assume \( \alpha = 1 \), \( \beta = 0.25 \), \( \phi = 1 \) and \( \chi = 1 \) in all scenarios.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Homogeneous</th>
<th>Heterogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( M )</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>( \eta )</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>( \theta )</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>( \delta^* )</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Figure 8 reports the result of the simulation in the homogeneous (first row) and in the heterogeneous (second row) case. Even if we consider random networks, the value \( \delta^* \) remains a relevant upper bound, because if \( \delta > \delta^* \) all banks fail and we have a systemic event irrespective of the network average degree. We then probe the model with \( \delta \) in the interval \((0, 0.1)\). On the x-axis and y-axis we report respectively the bank-firm average degree \( AD_{B,F} \) and the inter-bank average degree \( AD_{B,B} \). Low values of \( AD_{B,F} \) correspond to a poorly connected inter-bank, while higher values correspond to an highly connected network. The same applies for the values of \( AD_{B,B} \) in the inter-bank network. Different colors represent different levels of systemic risk: as shown by the vertical bar on the right-hand side of the heat-map, colors towards blue correspond to low levels, while colors toward red to high levels. Since we explore cases for \( \delta < \delta^* \), we know that if the connectivity of the network increases enough, the risk goes to zero. But for less connected network, the risk increases noticeably also for a relatively small fraction of NPLs. What seems to play the major role, at least for relatively large values of \( \delta \), is the bank-firm topology. This is not surprising as the average degree of the inter-bank is a measure of diversification of bank external assets. So we simply observe that more diversified banks are more resilient to an abrupt increase in NPLs. Conversely, the degree of connectivity in the bank-bank network appears to have a marginal role, at least for \( \delta \geq 0.4 \). If banks are not diversified enough in their exposition toward the risky firms, increasing the average degree of the inter-bank market will not save them from default.

We then move to the case of heterogeneous nodes. Each bank \( i = 1, \ldots, N \) is initialized with an amount of total bank assets \( A_{i}^{\text{TOT}} \) randomly drawn from a truncated power law distribution with bounded support \([5, 100]\) and exponent equal to 2.\(^{27}\) Similarly, to each firm \( j = 1, \ldots, M \) is assigned a value of total assets \( F_j \), distributed according to a truncated power law with support \([5, 100]\) and exponent 2. In this case, also to make the finer details of the models more clearly apparent, we focus on a range of values for the average degrees which is economically more reasonable, even if we probe a relatively large extent of possible values. In fact, the real value of \( AD_{B,B} \) and \( AD_{B,F} \) is in general not known and the few estimates of the inter-bank average degree present in the literature often refer only to short term lending. For example Anand et al. (2015) finds that the

\(^{27}\) The truncated power law with exponent \( \tau \) and support \([a, b]\) has a distribution function \( F(x) = (1 - a^{-\tau}x^{-\tau+1})/(1 - (b/a)^{-\tau}) \).
Figure 8: Level of systemic risk as function of the average degree in the inter-bank layer ($AD_{B,B}$) and in the bank-firm layer ($AD_{B,F}$). In the first row the homogeneous case, in the second row the heterogeneous case. The initial shock ($\delta$) is increasing from left to right. Estimates are obtained averaging over 200 independent realization of the model. The values of the parameters used in the simulations is reported in Table 2.
average degree for the German inter-bank network is 10.5, while Soramäki et al. (2007) finds an average degree of 15.2 for the Fedwire inter-bank payment network.

The four panels in Figure 8 show the level of systemic risk associated with different increases in the percentage of NPLs over total gross loans. The values of \( \delta \) tested go from 1.25% to 5%, well below the critical threshold \( \delta^* \). As can be seen, when heterogeneity is fully taken into consideration, the topology of the bank-bank network becomes more relevant. In fact, the figure shows that, on both the axes, the levels of systemic risk first increases and then decreases, showing a non-monotonic behaviour and peaking in the bottom-left area of all the panels. Notice that the scale of these plots is reduced. When \( AD_{B,B} \) and \( AD_{B,F} \) become larger the system converge to the fully connected case and the systemic risk goes to zero.\(^{28}\) Also notice that an higher initial shock does not change the general shape of the plot, but rather increases the overall level of risk for a maximum of 0.7 when \( \delta = 0.025 \) to a maximum of 1 when \( \delta = 0.05 \).

The role of the size of the initial NPLs shock can be better assessed by varying \( \delta \) while keeping fixed the average degrees of the inter-bank and inter-banks. Figure 9 shows the fraction of defaulted banks as a function of the level of the shock. In all the panels is evident the presence of a relatively steep phase transition, which implies that for certain level of NPLs, a small change in the magnitude of the initial shock can have very different consequences in terms of banks’ defaults. These transitions allow to identify a threshold level of \( \delta \), which is an important measure of the resilience of the system to external shocks. As expected this value is below \( \delta^* \) but the exact position of the transition varies slightly depending on the values of \( AD_{B,B} \) and \( AD_{B,F} \). Comparing the four panels of Figure 9 it is possible to see that, in line with the previous analysis, an increase in \( AD_{B,F} \) moves

\(^{28}\)Given the purely illustrative intent of the work we show only some selected charts in order not to overload the reading. A full set of charts is available upon request.
the transition to the right, making the banks more resilient to external shocks, while an increase of $AD_{B,B}$ makes the curve higher, thus enhancing the disruptive effect of bankruptcy cascades. It is worth highlighting that in the two top panels of Figure 9 the number of defaulted banks tends to 1 without reaching it. This is due to the fact that for very low values of the inter-bank average degree, some banks are completely disconnected and hence, if they survive the initial shock, they never fail since they cannot be reached by the contagion cascades.

5 Conclusions

In this paper we studied the implications of non-performing loans for financial stability using a network-based approach. We started by merging data from DealScan and Orbis in order to reconstruct the empirical financial network between banks and firms at global level in the period 1991-2016. We then identified a series of stylized facts which showed that: i) the international financial network has a scale-free structure, it is characterized by a highly right-skewed degree distributions (approximately a power law) and by a negative degree assortativity; ii) there is a positive correlation between the number of connections and the total assets of a node, so that bigger nodes are also more connected; iii) the amount involved in a financial transaction is proportionally related to the size of the two parties.

Based on our empirical findings we then developed a model in which two types of agents, banks and firms, are linked in a network by their reciprocal claims and analyze how an exogenous increase in non-performing loans affects the stability of the system.

From our analysis we found that the level of systemic risk varies with the level of interconnectedness in a non-monotonic way and that in order to effectively reduce the risk, banks should at the same time diversify their external portfolio and increase the number of their neighbours in the inter-bank market. In terms of resilience, we derived analytically the maximum increase of NPLs that a system of homogeneous and fully connected banks can bear without going bust. Shocks above that threshold lead to the collapse of the entire banking sector, which therefore exhibits an abrupt phase transition. Introducing heterogeneity complicates the analysis and does not allow to find a closed form solution for the critical level of the NPLs shock. We therefore relied on numerical simulations, which confirmed the existence of a phase transition and showed that, compared to the homogeneous case, heterogeneity weakens the system as it reduces the value of NPLs for which the critical threshold occurs. Although less sharp, the presence of a phase transition highlights how small variations in the magnitude of the initial shocks can have very different consequences in terms of number of defaults, so the effects of similar shocks can be hard to forecast based on previous experience as this may provide little guidance.

Moreover, while the capitalization level ($\eta$) and the intensity of the shock ($\delta$) have monotonic and unambiguous effects for the stability and resilience of the financial network, the level of diversification ($AD_{B,B}$ and $AD_{B,F}$) and the exposure of banks toward other banks ($\theta$) lead to more complex dynamics. This aspect is particularly relevant from a policy point of view: indeed while one can be sure that an higher $\eta^{29}$ and a lower $\delta$ are

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29Here we are clearly abstracting from any consideration regarding the provision of credit to the real economy in case of increased capital requirements.
always beneficial, it is not obvious what should be the desired level of $\theta$ as well as $AD_{B,B}$ and $AD_{B,F}$.

Two further considerations are in order. First, our simulations show that “intermediate” degrees of diversification are the most dangerous, while both very low and very high level of diversification would be preferable. An issue is whether such conditions can be attained in practice. Indeed in modern financial systems it is hard to imagine a bank which lends only to a very restricted circle of clients, but also a single bank that lend to an extremely large number of counter-parties. On top of this there is another aspect to bear in mind: in our model we did not investigate the rational behind the micro-behaviour of banks and in particular we did not specify any cost of link formation. However, in reality there are searching, screening and monitoring costs associated to each loan made, as well as rationing motives. In presence of limited resources, costs to form links and banks that try to maximise their profits, it is likely that the number of links generated will be neither too low nor too high, and so the degree of diversification toward which the system naturally tends is an “intermediate” one. In such a context, a policy aiming to move the system in a point of the “average-degree space” which is too distant form the one where it would tend seems hard to implement. The second consideration is about the role of $\theta$. As already mentioned the level of exposure toward other financial institutions has a non-monotonic effect of the stability of the system, but contrary to the degree of diversification it is much more controllable from policy makers. Therefore setting limits on the exposure between banks, or more generally among financial institutions, could be an easy and straightforward way to increase financial stability. It is interesting to note that the Glass-Steagall Act, approved in 1933 in the U.S. as a response to the crisis of 1929, was ultimately doing that.

While the model presented in the paper is admittedly very simple, in presence of detailed data on bilateral exposures between banks and firms it can contribute to assess the stability of the financial system. For example, with our model it would be possible to run counterfactual exercises to whether the level of NPLs in an economy is getting critical, or whether the level of capitalization of banks is too low, what is the most appropriate level of inter-bank exposure and so on.

The theoretical framework provided can be extended in several ways. First, the micro behaviour governing the process of network formation is certainly an interesting area of

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30The empirical evidence available seems indeed to suggest that financial networks are, as many other social networks, typically very sparse.

31Indeed, a part from separating commercial and investment banks and introducing a deposits’ guarantee, the Act limited the exposition of commercial banks within the inter-bank network by setting limits on the weights of their incoming links. In particular, the law imposed limits to banks on the quantity of securities (which are typically the tool that banks use to trade with each other when hedging against the risk that arises during their business) that they could hold, specifying maximum amounts both at issue level and at debtor level (see Section 16 of Glass-Steagall Act 1933).

32From a network point of view, the Glass-Steagall Act imposed limits on each link, which, added up at node’s level, corresponded to limits on $\theta$. The conditions set by the Glass-Steagall Act were therefore acting at loan level rather than at node level and were hence even more stringent than simply fixing the total value of $\theta$. Similar measures have been recently introduced also by the Federal Reserve (2015) and the Basel Committee’s (2014), which developed a standard to limit the exposures between banks’ up to 25% of the creditor bank’s capital, or 15% if the bank is systemically important (for a more detailed discussion see Glasserman & Young (2016)). However it still remains unclear whether the percentages chosen are “optimal” from a systemic point of view and in particular with respect to the level $\theta^*$. 

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research that would deserve deeper investigation. Second, in this paper we provided an indirect way to estimate a country’s financial network based on its economic structure and a number of additional assumptions. While our assumptions are based on empirically documented facts, more empirical analysis of whether the network generated by our algorithm is a good approximation of reality is certainly needed (and it could be facilitated by the growing amount of data on banks’ exposures collected by regulators at national and international level). Third, while our dataset contains information on loans at international level, our model abstracts from this dimension. Modeling cross-countries credit relationships would allow to take into account also the international propagation of shocks, which was crucial in spreading globally the crisis in 2007-2008. A relatively straightforward way to achieve this in our framework, is to draw from the gravity models developed in the trade literature and introduce a “distance parameter” in the linking functions, to capture the effect of geographic factors in forming links. In our view this is a promising and potentially fruitful direction of research which we leave for future work.
References


A DealScan-Orbis linking table

To construct the linking table we started by storing in two lists all the companies names present in DealScan and in Orbis. We then remove any character different from number, letter and single space from both lists. After we proceed as follows:

- match companies in the two databases which have exactly the same name;
- take the remaining firms and use the ticker to reduce the number of possible matches:
  - 1) match companies in the two databases which have exactly the same ticker;
  - 2) check if the names of the companies with the same ticker are the same by computing a measure of similarity that range from 0 to 100;
  - 3) if the similarity index is above 75% keep the match
- take all companies that have not yet been matched; proceed to a pure fuzzy match looping on every possible Orbis names; keep the best match (namely the match with the highest similarity index and in case of equal value of the index, the shortest Orbis name)
- visually inspect all the matches that have an index below than 75%
B Model application: a stress-testing exercise for Italy, Germany and United Kingdom

For illustrative purpose, in this section we provide an example of how the model could be applied in stress-tests. In particular, we calibrate the parameters with firm-level data and simulate the effects of a shock in the case of Italy, Germany and United Kingdom.

We choose 2013 as reference year and, for each country, we take information from Orbis on the value of $A^{IB}$, $A^{TOT}$, $K$ and $F$. Form these distributions we randomly sample without replacement to get the initial values of the total assets of banks and firms. We also compute the mode of $\eta$ and $\theta$ and assume it constant across all banks in each country. We then populate the balance sheet of the banks and generate the financial network as described in sections 4.2 and 4.3. Finally, we shock the system with an increase in NPLs, as in section 4.4.

More in detail, the data for this application come from two subsets of Orbis, Amadeus and Bankscope. Bankscope contains financial data on banks and other kinds of financial institutions worldwide. From the information available it is not possible to immediately identify the type of institution, therefore in order to extract a sub-sample consisting only of banks, we match the data from Bankscope with the list of authorized credit institutions (excluding branches) published by the European Banking Authority (EBA Register of Credit Institutions). Amadeus has financial data on European firms. We restrict our focus on firms with more than 50 employees, excluding NACE sectors 64.1 (Monetary intermediation) and 64.2 (Activities of holding companies), so as not to include the banks among other firms and to focus on firms whose activity is not just to own shares of other companies.

Bankscope and Amadeus report information about banks and firms at different consolidation levels. In order to avoid double counting issues and keep banking groups as much aggregated as possible, we selected data associated to consolidation codes $U1$, $C2$ or $C1$. After this procedure, and considering only banks and firms which have a value greater than 0 for all the variables considered, we finally have 1665 banks and 25855 firms for Germany, 527 banks and 23929 firms for Italy and 163 banks and 38521 firms.

However, we do not calibrate the value of the exponents of the linking functions because we do not have sufficient data for domestic lending in Italy, Germany and United Kingdom. We could estimate the exponent using global data, but then we would need to assume that the values are the same in all the three countries. Since this is likely not to be the case, we preferred to keep the values of the generic model and to explore other dimensions, such as the ratio of banks and firms. A more precise estimation is left for future work.

The choice of the year is ultimately arbitrary and linked to the availability and coverage of the data at our disposal.

Since we have the whole empirical distribution of $\eta$ and $\theta$ we could also randomly sample their values as we do for the other parameters. However, we decided not to do so since in our linking function these two variables do not play any role. Moreover, a fixed value of $\eta$ and $\theta$ across banks allows us to easily identify the value of $\delta^*$ and use it to guide our simulations (in particular it allows us to restrict the range of meaningful values for the shock). Finally, in practice the minimum level of $\eta$ is influenced by regulations and hence is rather homogeneous across banks.

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We do so both because the quality of data for firms with less than 50 employees is lower, both because small firms are likely to have small credit lines with banks and therefore a negligible impact.

See the guide provided by Bureau van Dijk for more information.
for U.K. Notice that, as pointed out by Duprey & LÉ (2016) in relation to Bankscope (but the same holds for Amadeus), even after having considered consolidated entities, it is still possible to incur in some double counting. This problem can be solved only with information about firm ownership for all the years corresponding to the date at which the observations have been recorded. These data are not available to us. In any case, for the purposes of the present work, essentially based on distributional properties, the possible occurrence of some double counting in the banks or firms sample is likely to have a small and negligible effect. However, we observed that the number of banks and firms in these national economies is too large to be simulated effectively. In order to maintain the same proportionality observed in real data, the number of nodes in the inter-bank sector is set to 100 and the number of firms proportionally adjusted.

Table 3 reports the adjusted number of banks and firms used in the simulations, as well as the values of $\eta$ and $\theta$ which have been computed as the modal values of the respective distributions. As for the high value of the estimate of the capitalization level in UK, this is roughly in line with the level of regulatory capital – Tier 1 and Tier 2 – reported by the Bank of England. The annual average of the banking sector regulatory capital is indeed about 17% in 2014, 18% in 2015 and 20% in 2016. Finally, the critical value $\delta^*$ is 9.1% for Germany, 9.8% for Italy, and 20.5% for UK. The relatively higher value for the latter is due to both a portfolio effect, with UK banks having a larger share of their portfolio invested in inter-bank assets, and to a leverage effect, with UK banks being more capitalized than Italian or German banks for the same level of total capital.

Figures 10 shows the results of the simulations in terms of systemic risk in the cases of Italy, Germany and United Kingdom.

For each country, we probed a range of values for the initial NPLs shock $\delta$ from zero to the respective $\delta^*$. As expected when the size of the initial shock approach the critical value, the risk becomes higher irrespective of the network structure. For lower values of $\delta$, the results obtained for the three countries are more “noisy” that the ones obtained for the heterogeneous case. Given the high firms to banks ratio, in Italy and UK the diversification aspect of banks portfolio has a central role. The average degree of the inter-bank is what mainly decides the level of risk, while the inter-bank network is less relevant. In Germany this ratio is lower and the topology of the inter-bank network has a more prominent role. What is common in all three cases is that the level of risk cannot

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Germany</th>
<th>Italy</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>M</td>
<td>1500</td>
<td>4500</td>
<td>23600</td>
</tr>
<tr>
<td>$\eta$</td>
<td>8.4%</td>
<td>9.2%</td>
<td>18.8%</td>
</tr>
<tr>
<td>$\theta$</td>
<td>7.8%</td>
<td>6.5%</td>
<td>8.6%</td>
</tr>
<tr>
<td>$\delta^*$</td>
<td>9.1%</td>
<td>9.8%</td>
<td>20.5%</td>
</tr>
</tbody>
</table>

Table 3: Values of the parameters used in the Monte Carlo simulations for the stress-test exercises. See Table 2 for more information.

Ownership data are provided by Bankscope, but they require an extra license; moreover ownership data are in the cross-section for the current years, therefore in order to get the evolution over time of ownership structure it is necessary to use the updated version of the database at that time. Unfortunately no value for 2013 is available as the reports of the Bank of England start in 2014. For more information visit http://www.bankofengland.co.uk/pra/Pages/regulatorydata/default.aspx
be, in general, effectively reduced increasing only the inter-bank or the bank-firm average degree. The most efficient strategy is always to have banks that are at the same time sufficiently diversified in their external lending and have sufficient connections with other banks.

In Figure 11 we report the fraction of defaulted banks for different levels of initial shock and different combinations of average degrees in the case of the three countries. As it can be seen, the overall effect of the average degree in the inter-bank and inter-bank is different for the different countries. Nevertheless, the same considerations made for the heterogeneous apply also to the other three cases, both in terms on non-monotonic behavior of the systemic risk (with levels and position varying in the $AD_{B,B}$, $AD_{B,F}$ space according to country characteristics) and for what regards the effects of an increase in the average degrees on the fraction of defaulted banks over the total number of banks. Moreover, heterogeneity reduces the amount of NPLs for which the critical threshold occurs, as the transition to the default of the entire financial system occurs for lower values of $\delta$ compared to the homogeneous fully-connected case.

Comparing the curves in Figure 11 Germany appears to be structurally the weakest of the three countries, as the phase transition from a situation of low distress to a situation of high distress occurs earlier (namely for lower values of $\delta$) than the other two countries, for all the combinations of average degrees. The United Kingdom appears to be the most resilient among the countries considered for all the levels of connectivity, while Italy places itself in the middle. These results reflect the structural characteristics of the different economies and are based on the assumption that all the three countries are exposed to the same exogenous shock. Clearly in practice the magnitude of the shock to which countries are exposed is different and in this light the weakest country is obviously Italy, as it is the only one of the three which experienced a sustained increase in the level of NPLs, while Germany and United Kingdom registered a decline to levels equals or lower than the pre-crisis period. To give and idea of the magnitude of the issue, in Italy just from December 2012 to December 2013, NPLs increased from 124.973 millions euro to 155.885 millions euro (+24,7%)

\[ \delta = 1.6\% \]

This is clearly a rough estimate, as it assumes that all the new NPLs come from loans made before the end of 2012 (a fraction of them could come from new loans made during 2013, although likely very small), but still it gives a sense of the magnitudes at play and it is also consistent with the evidence in the right panel of Figure 1.

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\[ \text{[42]Elaboration of the authors based on data from Bank of Italy and Italian banking association (ABI). For more information see link to the ABI website.} \]

\[ \text{[43]This is clearly a rough estimate, as it assumes that all the new NPLs come from loans made before the end of 2012 (a fraction of them could come from new loans made during 2013, although likely very small), but still it gives a sense of the magnitudes at play and it is also consistent with the evidence in the right panel of Figure 1.} \]
Figure 10: Level of systemic risk as function of the average degree in the inter-bank layer ($AD_{BR}$) and in the bank-firm layer ($AD_{BF}$). The first row refers to Germany, the second to Italy, the third to the United Kingdom. The initial shock ($\delta$) is increasing from left to right. Estimates are obtained averaging over 200 independent realizations of the model. The values of the parameters used in the simulations is reported in Table 3.
Figure 11: Fraction of defaulted banks for different levels of initial shock and different combinations of average degrees. The average degree is expressed as a fraction of bank nodes in the case of $AD_{B,B}$ and as a fraction of firm nodes in the case of $AD_{B,F}$. 