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The Optimized Monetary Policy ZLB Mandate in NK Behavioural and RE Models Compared

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Abstract

In this paper, we extend the basic 'workhorse' New Keynesian model by relaxing the rationality assumption in favor of bounded rationality. We broadly follow (Gabaix, 2020) whereby agents are unable to anticipate macroeconomic developments perfectly and assumed to be both partially myopic and inattentive. In my set-up, we assume that agents form beliefs over the future infinite time horizon of aggregate states and prices which are exogenous to their decisions. we then directly apply agents' inattentiveness on these exogenous market factors into their infinite-time-horizon decision rules. This approach produces identical decision rules as the *"sparse agent"* approach of Gabaix. Most importantly, with my method we can derive a fully non-linear form of my model, which is necessary to provide a correct welfare rankings in my mandate study. Results by the standard linear Bayesian estimation technique in this paper show that the boundedly rational expectation (BR) model outperforms the fully rational expectation model (RE). In the BR model, introducing the zero-lower-bound (ZLB) episode results in a higher welfare cost compared to the RE model. As a result, the optimal steady state inflation level of the BR model is higher given a probability of the nominal interest hitting the ZLB. Optimized interest rate rules for the RE and BR models closely mimicks a price-level targeting rule. However, under the RE model, the nominal interest rate reacts more aggressively to the price inflation.

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Contents

1 Introduction

It is now widely accepted that the rational expectations hypothesis falls short of capturing the complex networks of modern economies. But there is less agreement on the appropriate deviation from this hypothesis needed to investigate economic policy applications. Optimal monetary policy conclusions derived from a fully rational model may well have poor consequences if implemented in a non-rational world.

Following Gabaix (2020), this paper proposes one way to analyze monetary policy when agents are not fully rational. In particular, it enriches the basic the New Keynesian (NK) model of Woodford (2003) by incorporating behavioral factors. In the baseline NK model the agent is fully rational but here, in contrast, the agent are partially myopic and does not perfectly anticipate future events even in the absence of future shocks. Moreover the agent is inattentive with respect to contemporaneous events. Up to first order approximation, the formulation takes the form of a parsimonious generalization of the traditional model. However, in order to account for an accurate welfare ranking exercise we incorporate the assumption of myopic agents (see Gabaix (2020)) with an 'anticipated utility' approach described below to derive a non-linear setting model that allows for welfare-based analysis of monetary policy.

1.1 Literature

This paper is first related to a strand of the literature on boundedly rational expectation framework (Kreps (1998), Sims (2005), Woodford (2013) and Gabaix (2020)). In general, bounded rationality means agents are inattentive to exogenous variables of interest and future economic conditions. There have been important empirical studies pointing out that agents' expectations are boundedly rational (Coibion and Gorodnichenko (2015)). The majority of the existing literature on behavioural New Keynesian models employs Euler learning, in which agents' decisions are based on first order conditions to maximisation problems. In contrast to the rational expectations solution, in which model consistent expectations enter the first order conditions, Euler learning uses simple bounded rational predictors alongside knowledge of the form of the rational expectations solution. However, a more recent literature on BR models also considers the anticipated utility approach where agents follow an optimal decision rule conditional on their beliefs over aggregate states and prices that are exogenous to their decision variables.[1] This takes into account all information available to the agent, and involves forecasts of variables external to them. The departure of BR (Gabaix (2020) and Woodford (2018*b*)) from aforementioned forms of BR is the use of cognitive discounting, in

[1]The anticipated utility approach with infinite time horizons is also referred the *infinite-time horizon* approach (Deak et al., 2015). Bounded rationality of this form can be generalized to finite time horizons - see Woodford (2018*b*).

which agents are unable to fully understand the world, especially the events that are far into the future. In order to capture this behavior Gabaix (2020) assumes that agents, as they simulate the future, the impacts of noisy shocks vanish in the far enough future. As a result, the model converges to the steady state of the economy or the default model.

The final strand of literature relates to the robustness benefits of optimized simple rules, inflation and price-level targeting; (see, for example, Svensson, 1999; Schmitt-Grohe and Uribe, 2000; Woodford, 2003; Vestin, 2006; Levine et al., 2007; Reiss, 2009; Gaspar et al., 2010; Giannoni, 2014; Deak et al., 2019). These papers examine the good robustness, determinacy and stability properties of price-level targeting. Holden (2016) shows these benefits extend to a ZLB setting. A very recent literature describes these benefits in terms of “make-up" strategies for central banks and in particular the Federal Reserve; see Powell (2020), Svensson (2020). Under such strategies policymakers seek to redress past deviations of inflation from its target. Assuming a make-up rule enjoys credibility, undershooting (overshooting) the target will raise (lower) inflation expectations, lower (raise) the real interest rate and help to stabilize the economy. Inertial Taylor rules have by design the make-up feature as they commitment to a response of the nominal interest rate to a weighted average of past inflation with the weights increasing with the degree of inertia. “Average inflation targeting” is a variant that sets a rolling window of cumulative past deviations; a further variant sets an asymmetric target whereby the central bank responds to average inflation above and below the long-run target in a different way. Hebden et al. (2020) provide details of these different makeup strategies and analyze their effectiveness using the Federal Reserve US macroeconomic model. In this paper we study optimized inertial Taylor rules that are parameterized so as to encompass a simple form of price-level targeting .

1.2 Main Features and Findings

As in Deak et al. (2020), we propose a general mandate framework designed by the policymaker that consists of four components: *(i)* a welfare objective delegated to the central bank, *(ii)* a zero-lower-bound (ZLB) constraint on the monetary instrument, the nominal interest rate in the form of an unconditional low probability of hitting the ZLB, *(iii)* a form of the central bank's Taylor-type interest rate policy rule that directs it to respond deviations of endogenous target macroeconomic variables and *(iv)* a long-run steady state inflation target. A mandate with these four features makes the central bank goal-dependent, but instrument-independent as it remains free to the choose the strength of its response to the targets in the rule. This paper studies and compares the results of this mandate framework under RE and BR models.

Results by standard linear Bayesian estimation technique show that the estimated BR model outperforms the fully rational expectation model (RE). Under the BR model, the ZLB episode results in a higher welfare cost compared to the RE model. The BR model produces a much higher optimal steady state inflation level given a probability of the nominal interest hitting the ZLB. Optimized interest rate rules for the RE and BR models approximately mimic a price-level target rule. However, under the RE model, the nominal interest rate reacts more aggressively on the price inflation.

1.3 Roadmap

The rest of the paper is structured as follows. Section 2 sets out a non-linear version of the Gabaix boundedly rational (BR) model while the RE counterpart is presented in the appendices. Section 3 presents the estimation exercise and compares the estimation results of the BR and RE models. Section 4 presents the main application of the models on optimal monetary policy with the zero-lower bound nominal interest rate constraint. Section 5 concludes the paper.

2 The Gabaix Behavioural Model

As indicated above there are a large number of different ways of modelling bounded rationality in NK macroeconomic models.[2] In this paper we choose to focus on the model of Gabaix (2020) for a number of reasons. First it is a parsimonious generalization of the widely used work-horse NK model as for example set out in the Gali (2015) recent text-book. Second, it is encompassed by another recent and influential paper, Woodford (2018*a*). Finally, two important paradoxes are resolved: forward guidance is much less powerful than in the standard RE NK model resolving the “forward guidance puzzle" and a permanent rise in the nominal interest rate cases inflation to fall in the short-run, and rise in the long-run resolving the “Fisher paradox".

[2] See Jump and Levine (2018) for a recent survey.

The rest of this section first describes the idea of a sparse agent that lies at the centre of the Gabaix model, subsection 2.1. Then in sub-sections 2.2 and 2.3 we use this concept to derive the behavioural household decisions of the household and the price-setting firm. Sub-section 2.4 then sets out the linearized model about a zero net inflation steady state that recovers the linear model of Gabaix (2020). It is important here to emphasize that we employ a non-linear set-up with a non-zero net inflation rate in the steady state with a ZLB constraint, features that are essential for the optimized simple rules that follow.

2.1 The Sparse Agent

Gabaix (2020) generalizes the max operator in economics by assuming less than fully attentive agents. The general idea is as follows: The traditional agent will solve a standard maximization problem: i.e.:

$$Max_a u(a,z) \quad subject \quad to \quad b(a,z) \geq 0 \tag{1}$$

where u is the utility function and b is a constraint.

The *"sparse agent"* will then solve an attention augmented maximization problem as following:

$$SMax_a u(a,z,m) \quad subject \quad to \quad b(a,z,m) \geq 0 \tag{2}$$

where $m \in [0,1]$ is a vector of agent's attention degree. The idea of a *"sparse agent"* is that she has a low-dimensional sub-model of the world. Hence, first she pays attention only to a few dimensions of the world - which is usually endogenously determined by assuming that attention creates a psychic cost function - or the attention vector is sparse, and second she takes decisions by optimizing her sub-model of the world.

In the concept of this paper, we assume that the agent's attention degree level is exogenously determined, or the attention parameter vector, m, is given, which means the Sparsemax operator is simplified as the standard maximization operator while the only difference is in the attention vector of parameter m. This attention parameter vector will then be matched with the data by standard Bayesian estimation.

In the Section 2.4 we show that with an exact households' utility function and firms' production function, we can derive log-linearized version of the IS and Phillips curves by solving the model forward and then directly apply this inattentive vector m into households' and firms' decision rules.

2.2 Household Decisions

In this section, there are three distinct features compared to the one in Gabaix (2020). First, we employ the log form in consumption of the utility function (or $\gamma = 1$). Second, instead of assuming that households are only inattentive to their total income, we distinguish between wage's and government transfer's incomes. Therefore, households' inattentive levels to these different income sources would end up being unequal. Third, we employ a non-linear set-up with an anticipated utility approach (as reviewed in Section 1.1) with a non-zero net trend inflation rate.

Household j chooses between work and leisure and therefore how much labour they supply and how much she consumes today. Let $C_t(j)$ and $H_t(j)$ denote consumption and labour supply, respectively. The single-period utility is given by

$$U_t(j) = U(C_t(j), H_t(j)) = \log(C_t(j)) - \kappa \frac{H_t(j)^{1+\phi}}{1+\phi} \tag{3}$$

In a stochastic environment, the value function of the representative household at time t is given by

$$V_t(j) = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U_{t+s}(j) \right] \tag{4}$$

The household's problem at time t is to choose paths for consumption $\{C_t(j)\}$, labour supply $\{H_t(j)\}$ and holdings of financial assets $\{B_t(j)\}$ to maximize $V_t(j)$ given by (72) given its budget constraint in period t

$$B_t(j) = R_t B_{t-1}(j) + W_t H_t(j) + \Gamma_t - C_t(j) - T_t \tag{5}$$

where $B_t(j)$ is holdings of financial assets at the end of period t, W_t is the real wage rate, R_t is the interest rate paid on assets held at the beginning of period t, Γ_t are profits from wholesale and retail firms owned by households and T_t denote taxes. W_t, R_t, Γ_t and T_t are all exogenous to household j.

To solve the household problem we form a Lagrangian which is presented in the Appendix where the full rational expectation model is also solved and presented.

For households, aggregating over j, we stationarize the non-stationary variables as follows:

$$\frac{B_t}{\bar{A}_t} = R_t \frac{B_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t} + \frac{W_t H_t}{\bar{A}_t} + \frac{\Gamma_t}{\bar{A}_t} - \frac{T_t}{\bar{A}_t} - \frac{C_t}{\bar{A}_t} \tag{6}$$

Solving it forward in time and imposing the transversality condition we can write:

$$\begin{aligned} B^c_{t-1} = {} & \mathbb{E}^{BR}_t \sum_{i=0}^{\infty} \frac{(1+g)^{i+1} C^c_{t+i}}{R_{t,t+i}} + \mathbb{E}^{BR}_t \sum_{i=0}^{\infty} \frac{(1+g)^{i+1} T^c_{t+i}}{R_{t,t+i}} \\ & - \mathbb{E}^{BR}_t \sum_{i=0}^{\infty} \frac{(1+g)^{i+1} W^c_{t+i} H_{t+i}}{R_{t,t+i}} - \mathbb{E}^{BR}_t \sum_{i=0}^{\infty} \frac{(1+g)^{i+1} \Gamma^c_{t+i}}{R_{t,t+i}} \end{aligned} \tag{7}$$

where $R_{t,t+1} \equiv R_t R_{t+1} R_{t+2} \cdots R_{t+i}$ is the real interest rate over the interval $[t, t+i]$. And the variables with superscript c are the stationary version of the endogenous variables, $X^c_t = \frac{X_t}{\bar{A}_t}$.

The forward-looking budget constraint (7) holds for the representative household. In aggregate there is no net debt so $B_{t-1} = 0$. Then in a symmetric equilibrium, substituting for

$$W^c_{t+i} H_{t+i} = \frac{(W^c_{t+i})^{1+\frac{1}{\phi}}}{(\kappa C^c_{t+i})^{\frac{1}{\phi}}} \tag{8}$$

Which is the first order condition on the household's supply decision with the following utility function:

$$U(C_t, H_t) = log(C_t) - \kappa \frac{H_t^{1+\phi}}{1+\phi}$$

From (7), substituting (8) and multiplying both sides by $R_t/(1+g)$ we have

$$\begin{aligned} \mathbb{E}^{BR}_t \sum_{i=0}^{\infty} \frac{(1+g)^i C^c_{t+i}}{R_{t+1,t+i}} = {} & \frac{(W^c_t)^{1+\frac{1}{\phi}}}{(\kappa C^c_t)^{\frac{1}{\phi}}} + \mathbb{E}^{BR}_t \sum_{i=1}^{\infty} \frac{(1+g)^i (W^c_{t+i})^{1+\frac{1}{\phi}}}{(\kappa C^c_{t+i})^{\frac{1}{\phi}} R_{t+1,t+i}} \\ & + \Gamma^c_t + \mathbb{E}^{BR}_t \sum_{i=1}^{\infty} \frac{(1+g)^i \Gamma^c_{t+i}}{R_{t+1,t+i}} - \mathbb{E}^{BR}_t \sum_{i=0}^{\infty} \frac{(1+g)^i T^c_{t+i}}{R_{t+1,t+i}} \end{aligned} \tag{9}$$

Solving the Euler equation $\frac{1}{C^c_t} = \left(\frac{\beta}{1+g}\right) \mathbb{E}^{BR}_t \left[\frac{R_{t+1}}{C^c_{t+1}}\right]$ forward in time we have for $i \geq 1$

$$\frac{1}{C^c_t} = \left(\frac{\beta}{1+g}\right)^i \mathbb{E}^{BR}_t \left[\frac{R_{t+1,t+i}}{C^c_{t+i}}\right] \; ; \; i \geq 1 \tag{10}$$

We assume point expectations, i.e. $E_t f(X_t) \approx f(E_t(X_t))$ and $E_t f(X_t Y_t) \approx f(E_t(X_t)E_t(Y_t))$. For instance, agents are only able to make single variable expectation rather than the expectation of the complicated functions. The concept of BR studied in this paper is about the limited cognitive capacities of the agents. This additional point expectation assumption is crucial to the result of the non-linear set-up, but it is in line with the agents' cognitive discounting assumption. Notice that, up to the first order Taylor approximation, this assumption is equivalent to using linear approximation as shown in the appendices, where the linear approximation set-up is equivalent to the linear set-up in Gabaix (2020).

We now rearrange (10) to obtain

$$\mathbb{E}_t^{BR} C_{t+i}^c \quad = \quad C_t^c \left(\frac{\beta}{1+g}\right)^i \mathbb{E}_t^{BR} R_{t+1,t+i} \ ; \ i \geq 1 \tag{11}$$

Using it on the the left-hand side of (9) we get

$$\begin{aligned}\sum_{i=0}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} C_{t+i}^c}{\mathbb{E}_t^{BR} R_{t+1,t+i}} &= \sum_{i=0}^{\infty} \frac{(1+g)^i C_t^c \left(\frac{\beta}{1+g}\right)^i \mathbb{E}_t^{BR} R_{t+1,t+i}}{\mathbb{E}_t^{BR} R_{t+1,t+i}} \\ &= \frac{C_t^c}{1-\beta}\end{aligned} \tag{12}$$

Using it on the right-hand side of (9) we get

$$\begin{aligned}\sum_{i=1}^{\infty} \frac{(1+g)^i (\mathbb{E}_t^{BR} W_{t+i}^c)^{1+\frac{1}{\phi}}}{(\kappa \mathbb{E}_t^{BR} C_{t+i}^c)^{\frac{1}{\phi}} \mathbb{E}_t^{BR} R_{t+1,t+i}} &= \sum_{i=1}^{\infty} \frac{(1+g)^i (\mathbb{E}_t^{BR} W_{t+i}^c)^{1+\frac{1}{\phi}}}{\left(\kappa C_t^c \left(\frac{\beta}{1+g}\right)^i \mathbb{E}_t^{BR} R_{t+1,t+i}\right)^{\frac{1}{\phi}} \mathbb{E}_t^{BR} R_{t+1,t+i}} \\ &= \left(\frac{1}{\kappa C_t^c}\right)^{\frac{1}{\phi}} \sum_{i=1}^{\infty} \frac{\beta^{-\frac{i}{\phi}} (1+g)^{i\left(1+\frac{1}{\phi}\right)} (\mathbb{E}_t^{BR} W_{t+i}^c)^{1+\frac{1}{\phi}}}{(\mathbb{E}_t^{BR} R_{t+1,t+i})^{1+\frac{1}{\phi}}}\end{aligned} \tag{13}$$

Substituting back into the forward-looking household budget constraint we arrive at

$$\begin{aligned}\frac{C_t^c}{1-\beta} &= \frac{(W_t^c)^{1+\frac{1}{\phi}}}{(\kappa C_t^c)^{\frac{1}{\phi}}} + \left(\frac{1}{\kappa C_t^c}\right)^{\frac{1}{\phi}} \sum_{i=1}^{\infty} \frac{\beta^{-\frac{i}{\phi}} (1+g)^{i\left(1+\frac{1}{\phi}\right)} (\mathbb{E}_t^{BR} W_{t+i}^c)^{1+\frac{1}{\phi}}}{(\mathbb{E}_t^{BR} R_{t+1,t+i})^{1+\frac{1}{\phi}}} \\ &\quad + \Gamma_t^c + \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} \Gamma_{t+i}^c}{\mathbb{E}_t^{BR} R_{t+1,t+i}} - T_t^c - \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} T_{t+i}^c}{\mathbb{E}_t^{BR} R_{t+1,t+i}} \\ &= \frac{(W_t^c)^{1+\frac{1}{\phi}} + \sum_{i=1}^{\infty} \left(\frac{(1+g)^{1+\frac{1}{\phi}}}{\beta^{\frac{1}{\phi}}}\right)^i \left(\frac{\mathbb{E}_t^{BR} W_{t+i}^c}{\mathbb{E}_t^{BR} R_{t+1,t+i}}\right)^{1+\frac{1}{\phi}}}{[\kappa C_t^c]^{\frac{1}{\phi}}}\end{aligned} \tag{14}$$

$$+\Gamma_t^c + \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} \Gamma_{t+i}^c}{\mathbb{E}_t^{BR} R_{t+1,t+i}} - T_t^c - \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} T_{t+i}^c}{\mathbb{E}_t^{BR} R_{t+1,t+i}} \tag{15}$$

Notice that from the (14) to (15), we use the notation such that $R_{t+1,t} = 1$.

Employing the Fisher relation, we obtains:

$$\mathbb{E}_t^{BR} R_{t+1,t+i} = \mathbb{E}_t^{BR} \left[R_{t+1} R_{t+2} \cdots R_{t+i} \right] = \mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right] \tag{16}$$

Substituting equation (16) into equation (15) to yields:

$$\frac{C_t^c}{1-\beta} = \frac{(W_t^c)^{1+\frac{1}{\phi}} + \sum_{i=1}^{\infty} \left(\frac{(1+g)^{1+\frac{1}{\phi}}}{\beta^{\frac{1}{\phi}}} \right)^i \left(\frac{\mathbb{E}_t^{BR} W_{t+i}^c}{\mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right]} \right)^{1+\frac{1}{\phi}}}{\left[\kappa C_t^c \right]^{\frac{1}{\phi}}}$$

$$+\Gamma_t^c + \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} \Gamma_{t+i}^c}{\mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right]} - T_t^c - \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} T_{t+i}^c}{\mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right]} \tag{17}$$

Hence, we can rewrite equation (17) as follows:

$$\frac{C_t^c}{1-\beta} = \frac{Z_t}{\left[\kappa C_t^c \right]^{\frac{1}{\phi}}} + ZZ_t \tag{18}$$

Where Z_t and ZZ_t are expressed as follows:

$$Z_t = (W_t^c)^{1+\frac{1}{\phi}} + \sum_{i=1}^{\infty} \left(\frac{(1+g)^{1+\frac{1}{\phi}}}{\beta^{\frac{1}{\phi}}} \right)^i \left(\frac{\mathbb{E}_t^{BR} W_{t+i}^c}{\mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right]} \right)^{1+\frac{1}{\phi}} \tag{19}$$

$$ZZ_t = \Gamma_t^c - T_t^c + \sum_{i=1}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} \Gamma_{t+i}^c}{\mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right]} - \sum_{i=0}^{\infty} \frac{(1+g)^i \mathbb{E}_t^{BR} T_{t+i}^c}{\mathbb{E}_t^{BR} \left[\frac{R_{n,t,t+i-1}}{\Pi_{t+1,t+i}} \right]} \tag{20}$$

We can now write equations (19) and (20) in a recursive form as follows:

$$Z_t = (W_t^c)^{1+\frac{1}{\phi}} + \left(\frac{(1+g)^{1+\frac{1}{\phi}}}{\beta^{\frac{1}{\phi}}} \right) \left(\frac{\mathbb{E}_t^{BR} Z_{t+1}}{\left[\frac{R_{n,t}}{\mathbb{E}_t^{BR} \Pi_{t+1}} \right]^{1+\frac{1}{\phi}}} \right) \tag{21}$$

$$ZZ_t = \Gamma_t^c - T_t^c + (1+g) \frac{\mathbb{E}_t^{BR} ZZ_{t+1}}{\left[\frac{R_{n,t}}{\mathbb{E}_t^{BR} \Pi_{t+1}} \right]} \tag{22}$$

We now follow Gabaix (2020) to assume that the behavioural agent perceives reality with some myopia, which is associated with deviations from the steady state of nominal interest rate, $\hat{R}_{n,t} = R_{n,t} - R_n$, gross expectation, $\hat{\Pi} = \Pi_t - \Pi$, $\hat{\Gamma}^c_t - \hat{T}^c_t = (\Gamma^c_t - \Gamma^c) - (T^c_t - T^c)$. Specifically, $R_{n,t} = R_n + m_{hr}\hat{R}_{n,t}$, $\Pi_t = \Pi + m_{h\pi}\hat{\Pi}_t$, $\Gamma^c_t - \hat{T}^c_t = \Gamma^c - T^c + m_{hy}(\hat{\Gamma}^c_t - \hat{T}^c_t)$. In addition, behavioural agent has to form expectation about the future state of the economy, say $E^{BR}_t X_{t+1} = \bar{m}_h f(X_t)$. Note that, households do not have inattentive level on the aggregate wage because the information on wage is available to households in the current period which comes straight into the optimality condition of labor supply. Hence, rewriting equations (21) and (22) yields:

$$Z_t = (W^c_t)^{1+\frac{1}{\phi}} + \left(\frac{(1+g)^{1+\frac{1}{\phi}}}{\beta^{\frac{1}{\phi}}}\right)\left(\frac{\mathbb{E}_t\left(Z + \bar{m}_h\hat{Z}_{t+1}\right)}{\mathbb{E}_t\left(\frac{R_n + m_{hr}\hat{R}_{n,t}}{\Pi + \bar{m}_h m_{h\pi}\hat{\Pi}_{t+1}}\right)^{1+\frac{1}{\phi}}}\right) \tag{23}$$

$$ZZ_t = \left(\Gamma^c - T^c + m_{hy}(\hat{\Gamma}^c_t - \hat{T}^c_t)\right) + (1+g)\left(\frac{\mathbb{E}_t\left(ZZ + \bar{m}_h\hat{ZZ}_{t+1}\right)}{\mathbb{E}_t\left(\frac{R_n + m_{hr}\hat{R}_{n,t}}{\Pi + \bar{m}_h m_{h\pi}\hat{\Pi}_{t+1}}\right)}\right) \tag{24}$$

The three equations (18),(23), and (24) then constitute a nonlinear behavioural *consumption function* which also nests the fully rational expectation one when the vector of inattentive parameters, m, is equal to a vector of ones. In general, equations (23), and (24) are the future discounted values of the proportioned wages and net transfers from the government to households. Hence, equation (18) shows that consumption today is an increasing function of future discounted values of household's incomes, but it is decreasing in the central bank's policy rates, $\frac{R_n + m_{hr}\hat{R}_{n,t}}{\Pi + \bar{m}_h m_{h\pi}\hat{\Pi}_{t+1}}$. Since these variables are exogenous to the atomistic household we therefore have an 'anticipated utility' form of household behaviour suitable for either our behavioural or RE models.

2.3 Price-setting Firms

The homogeneous production technology in the economy is:

$$Y^c_t = A^c_t H^\alpha_t \tag{25}$$

There is a probability of $1 - \xi$ at each period that the price of each retail good i is set optimally to $P^0_t(i)$; otherwise it is held fixed.

Retail behavioural producer i, given the common real marginal cost $MC_t(i) = MC_t$ chooses $\{P_t^0(i)\}$ to maximize discounted real profits

$$\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}^c(i) \left[P_t^0(i) - P_{t+k} MC_{t+k}\right] \tag{26}$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor over the interval $[t, t+k]$, subject to

$$Y_{t+k}^c(i) = \left(\frac{P_t^O(i)}{P_{t+k}}\right)^{-\zeta} Y_{t+k}^c \tag{27}$$

The solution to this is

$$\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} Y_{t+k}^c(i) \left[\frac{P_t^0(i)}{P_{t+k}} - \frac{1}{(1-1/\zeta)} MC_{t+k}\right] = 0 \tag{28}$$

which leads to

$$\frac{P_t^0(m)}{P_t} = \frac{1}{1-1/\zeta} \frac{\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta} Y_{t+k}^c MC_{t+k}}{\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta-1} Y_{t+k}^c} \tag{29}$$

where k periods ahead inflation is defined by

$$\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t} = \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} \cdot \cdot \frac{P_{t+k}}{P_{t+k-1}} = \Pi_{t+1} \Pi_{t+2} \cdot \cdot \Pi_{t+k}$$

Note that $\Pi_{t,t+1} = \Pi_{t+1}$ and $\Pi_{t,t} = 1$.

Let us define

$$\begin{aligned} J_t^c &= \frac{1}{1-\frac{1}{\zeta}} \mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\zeta} Y_{t+k}^c MC_{t+k} \\ &= \frac{1}{1-\frac{1}{\zeta}} Y_t^c MC_t + \xi \mathbb{E}_t^{BR} \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} J_{t+1}^c \end{aligned} \tag{30}$$

$$\begin{aligned} JJ_t^c &= \mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\zeta-1} Y_{t+k}^c \\ &= Y_t^c + \xi \mathbb{E}_t^{BR} \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} JJ_{t+1}^c \end{aligned} \tag{31}$$

Then (87) can be written as

$$\frac{P_t^0(m)}{P_t} = \frac{J_t}{JJ_t} \tag{32}$$

By the law of large numbers the evolution of the price index is given by

$$P_{t+1}^{1-\zeta} = \xi P_t^{1-\zeta} + (1-\xi)(P_{t+1}^0)^{1-\zeta} \tag{33}$$

which can be written as

$$1 = \xi \Pi_t^{\zeta-1} + (1-\xi)\left(\frac{J_t^c}{JJ_t^c}\right)^{1-\zeta} \tag{34}$$

We first transform the equations (88), (89) to the expectations of the **behavioural** agents, where we also utilise the relation $\mathbb{E}_t^{BR}\Lambda_{t,t+k} = \frac{1}{\mathbb{E}_t^{BR}R_{t+1,t+k}} = \frac{1}{\mathbb{E}_t^{BR}\left[\frac{R_{n,t,t+k-1}}{\Pi_{t+1,t+k}}\right]}$, and employing the assumption about the firms' myopia about the future state such that: $\mathbb{E}_t^{BR}(X_{t+1}-X) = \bar{m}_f\mathbb{E}_t(X_{t+1}-X)$. In addition, firms are inattentive to the market's variables which are exogenous to them. Hence, we can re-write the equations (88) and (89) in a recursive form as follows:

$$J_t^c = \frac{1}{1-\frac{1}{\zeta}}(Y_t^c)(MC + m_{fmc}\hat{MC}_t) + \xi\mathbb{E}_t\frac{(\Pi + \bar{m}m_{f\pi}\hat{\Pi}_{t+1})^{\zeta}}{\mathbb{E}_t\left(\frac{R_n + m_{fr}\hat{R}_{n,t}}{\Pi + \bar{m}_f m_{f\pi}\hat{\Pi}_{t+1}}\right)}(J^c + \bar{m}_f\hat{J}_{t+1}^c) \tag{35}$$

$$JJ_t^c = Y_t^c + \xi\mathbb{E}_t\frac{(\Pi + \bar{m}m_{f\pi}\hat{\Pi}_{t+1})^{\zeta-1}}{\mathbb{E}_t\left(\frac{R_n + m_{fr}\hat{R}_{n,t}}{\Pi + \bar{m}_f m_{f\pi}\hat{\Pi}_{t+1}}\right)}(JJ^c + \bar{m}_f\hat{JJ}_{t+1}^c) \tag{36}$$

As for the household, price-setting is now expressed in terms of real marginal cost and aggregate demand, variables that are exogenous to the atomistic firm. Again we therefore have an 'anticipated utility' form of firm behaviour suitable for either our behavioural or RE models.

2.4 Linearized Model

This sub-section shows that a special case of a linear approximation about the steady state of my non-linear set-up reduces to the log-linear formulation of Gabaix (2020). The special case is that of non-growth zero net inflation steady state. In addition we reformulate the budget constraint in line with Gabaix (2020) so that household is inattentive to total income $Y_t^h = W_tH_t + \Gamma_t - T_t$ Then the budget constraint (5) becomes:

$$B_t = R_tB_{t-1} + Y_t^h - C_t \tag{37}$$

The household first order solutions solution with respect to C_t and H_t are respectively:

$$\begin{aligned} C_t^{-\gamma} &= \beta \mathbb{E}_t^{BR}\left[R_{t+1}C_{t+1}^{-\gamma}\right] && (38)\\ W_t &= \frac{H_t^{\sigma}}{C_t^{-\gamma}} && (39)\end{aligned}$$

Solving the budget constraint (37) forward in time and imposing the transversality condition we can write:

$$B_{t-1} = \mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{C_{t+i}}{R_{t,t+i}} - \mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{Y_{t+i}^{h}}{R_{t,t+i}} \tag{40}$$

Hence, we can write:

$$\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{C_{t+i}}{R_{t,t+i}} = \mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{Y_{t+i}^{h}}{R_{t,t+i}} \tag{41}$$

Multiplying both sides of equation above with R_t, we get:

$$\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{C_{t+i}}{R_{t+1,t+i}} = \mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{Y_{t+i}^{h}}{R_{t+1,t+i}} \tag{42}$$

Assuming point expectations as before, solving the Euler equation (38) forward yields:

$$C_t^{-\gamma} = \beta^{i}\mathbb{E}_t^{BR}\left[R_{t+1,t+i}C_{t+i}^{-\gamma}\right]; \quad i \geq 1 \tag{43}$$

Or equivalently,

$$C_{t+i} = C_t\left[\beta^{\frac{i}{\gamma}}\mathbb{E}_t^{BR}R_{t+1,t+i}\right]^{\frac{1}{\gamma}}; \quad i \geq 1 \tag{44}$$

We next substitute equation (44) into equation (42):

$$C_t = \frac{\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{Y_{t+i}^{h}}{R_{t+1,t+i}}}{\mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\left[\beta^{\frac{i}{\gamma}}R_{t+1,t+i}^{\frac{1}{\gamma}-1}\right]} = \frac{Z_t}{ZZ_t} \tag{45}$$

where:

$$Z_t = \mathbb{E}_t^{BR}\sum_{i=0}^{\infty}\frac{Y_{t+i}^{h}}{R_{t+1,t+i}} \tag{46}$$

$$ZZ_t = \mathbb{E}_t^{BR} \sum_{i=0}^{\infty} \left[\beta^{\frac{i}{\gamma}} R_{t+1,t+i}^{\frac{1}{\gamma}-1} \right] \tag{47}$$

The behavioural household maximizes the same value function as before but does not pay full attention to all the variables in the budget constraint (37), as correctly processing information entails a cost. The behavioural agent perceives reality with some myopia, which is associated with deviations from the steady state real interest rate, $\hat{R}_t = R_t - R$, and real income, $\hat{Y}_t^h = Y_t^h - Y^h$. Specifically, $Y_t^h = Y^h + m_y \hat{Y}_t^h$ and $R_t = R + m_r \hat{R}_t$. In addition, behavioural agent has to form expectation about the future state of the economy, say $X_{t+1} = \bar{m} f(X_t)$. Hence, rewriting equations (46) and (47) yields:

$$Z_t = (C + m_y \hat{C}_t) + \mathbb{E}_t \left[\frac{Z + \bar{m} \hat{Z}_{t+1}}{R + m_r \hat{R}_{t+1}} \right] \tag{48}$$

$$ZZ_t = 1 + \beta^{\frac{1}{\gamma}} \mathbb{E}_t \left[\left(R + m_r \hat{R}_{t+1} \right)^{\frac{1}{\gamma}-1} (ZZ + \bar{m} \hat{ZZ}_{t+1}) \right] \tag{49}$$

Notice that, we also imposed the market clearing condition, $Y^h = C$ and $\hat{Y}_t^h = \hat{C}_t$, to derive equations (48) and (49).

Where the vector of $m = [\bar{m}, m_r, m_y] \in [0, 1]$ is the set of inattention parameters of the household to the exogenous variable and the future state of the economy. From equations (45), (48), and (49). When the vector of inattention parameters is equal to $\mathbf{1}$, we can easily retrieve the full-rational household's non-linear consumption function in the form of equation (38).

We now perform a standard log-linearization of the consumption function. First log-linearizing (48) and (49) around the steady stead $\frac{C}{Z} = 1 - \frac{1}{R}$gives

$$\tilde{Z}_t = m_y (1 - \frac{1}{R}) \tilde{C}_t + \frac{\bar{m}}{R} \tilde{Z}_{t+1} - m_r \frac{1}{R^2} \hat{R}_{t+1} \tag{50}$$

$$\tilde{ZZ}_t = \frac{\bar{m}}{R} (\beta R)^{\frac{1}{\gamma}} \tilde{ZZ}_{t+1} + m_r (\beta R)^{\frac{1}{\gamma}} (\frac{1}{\gamma} - 1) \frac{1}{R^2} \hat{R}_{t+1} \tag{51}$$

where $(\beta R)^{\frac{1}{\gamma}} = 1$ and log-linearising equation (45) yields $\tilde{C}_t = \tilde{Z}_t - \tilde{ZZ}_t$. hence, we can subtract equation (50) by equation (51) to get:

$$\tilde{C}_t = \tilde{Z}_t - \tilde{ZZ}_t = m_y (1 - \frac{1}{R}) \tilde{C}_t + \frac{\bar{m}}{R} (\tilde{Z}_{t+1} - \tilde{ZZ}_{t+1}) - \frac{m_r}{\gamma R^2} \hat{R}_{t+1} \tag{52}$$

which is gives the linearised consumption function as in Gabaix (2020):

$$\tilde{C}_t = \frac{\bar{m}}{[R - m_y(R-1)]}\tilde{C}_{t+1} - \frac{m_r}{\gamma R[R - m_y(R-1)]}\hat{R}_{t+1} \tag{53}$$

Turning to the Phillips curve we log-linearise equations (35), (36), and (34) again conditional on the zero growth and net inflation steady state inflation to get:

$$\tilde{J}_t = (1 - \beta\xi\Pi^{\zeta})\left(\tilde{Y}_t + m_{fmc}\tilde{MC}_t\right) + \beta\xi\Pi^{\zeta}\mathbb{E}_t\left((1+\zeta)\bar{m}_f m_{f\pi}\tilde{\Pi}_{t+1} - m_{fr}\tilde{Rn}_t + \bar{m}_f\tilde{J}_{t+1}\right) \tag{54}$$

$$\tilde{JJ}_t = (1 - \beta\xi\Pi^{\zeta-1})\tilde{Y}_t + \beta\xi\Pi^{\zeta-1}\mathbb{E}_t\left(\zeta\bar{m}_f m_{f\pi}\tilde{\Pi}_{t+1} - m_{fr}\tilde{Rn}_t + \bar{m}_f\tilde{JJ}_{t+1}\right) \tag{55}$$

$$\tilde{\Pi}_t = \frac{1 - \xi\Pi^{\zeta-1}}{\xi\Pi^{\zeta-1}}(\tilde{J}_t - \tilde{JJ}_t) \tag{56}$$

Notice that the expectation terms here are fully rational, the vector of myopia parameters included in the set of equations above represents the behavioural element of the boundedly rational price-setting firms. When the steady state of inflation is zero (or the steady state gross inflation $\Pi = 1$), we can directly subtract equation (55) from equation (54) and substitute into equation (56) to eliminate $\tilde{Y}_t$ and $\tilde{Rn}_t$ to get a standard Phillips curve at the zero steady state level of inflation as follows:

$$\tilde{\Pi}_t = \frac{(1-\xi)(1-\beta\xi)m_{fmc}}{\xi}\tilde{MC}_t + \beta\left[(1-\xi)\bar{m}_f m_{f\pi} + \xi\bar{m}_f\right]\mathbb{E}_t\tilde{\Pi}_{t+1} \tag{57}$$

Again, we can retreat the Phillips curve of the fully rational price-setting firm if the vector of myopia parameters, $[\bar{m}, m_{fmc}, m_{f\pi}, m_{fr}]$, is equal to the vector of 1. Although my behavioural Phillips curve (with zero steady state inflation) is isomophic to that of Gabaix (2020), my behavioural Phillips curve also has the same property as Gabaix's which is less forward-looking compared to the fully rational case. In other word, when firms are more attentive to the macroeconomic outcomes, say, vector m is closer to one, then firms are more forward-looking because the slope on future inflation is higher.

3 Bayesian Estimation

This section sets out the Bayesian estimation of the model using the same standard techniques as Deak et al. (2020). The model is linearized computationally about the non-net inflation positive growth deterministic state set out in Appendix B.1. Before presenting the results, we first describe the measurement equations and the data. The methodology and identification follow as in Deak et al. (2020). We again highlight the information assumptions made in solving for a RE equilibrium that are usually only implicit in Bayesian estimation exercises.

3.1 Data and Measurement Equations

My observables used in the estimation are: GDP per capita growth (`dyobs`), percentage deviation of hours worked and supplied per capita from mean (`labobs`), the monetary policy rate (`robs`) and the inflation rate (`pinfobs`). The corresponding measurement equations are:

$$\begin{aligned}
\texttt{dyobs} &= \log\left((1+g)\frac{Y_t^c}{Y_{t-1}^c}\right) && (58)\\
\texttt{labobs} &= \frac{H_t^d - H^d}{H^d} && (59)\\
\texttt{robs} &= R_{n,t} - 1 && (60)\\
\texttt{pinfobs} &= \log(\Pi_t) && (61)
\end{aligned}$$

The steady state values of the observables are `dyobs` $== \log(1+g)$, `labobs` $= 0$, `robs` $= R_n - 1$, and `pinfobs` $= log(\Pi)$.

The original data are for these variables is the same as Deak et al. (2020) is taken from the FRED Database available through the Federal Reserve Bank of St.Louis. The data consists of 4 quarterly time series, namely log output growth ($dyobs$), labour hours supply ($labobs$), the net inflation ($pinfobs$), and finally the policy rate measurement ($robs$). The sample period is again 1958:1-2017:4. There is a pre-sample period of 4 quarters so the observations actually used for the estimation go from 1959:1-2017:4, 240 observations.

Parameters		Prior		Post. RE			Post. Behaviour		
	pdf	Mean	Std	Mean	5 %	95 %	Mean	5 %	95 %
Technology shock (ϵ_a)	**IG**	0.001	0.02	0.0065	0.0060	0.0069	0.0065	0.0060	0.0070
Mark-up shock (ϵ_{ms})	**IG**	0.001	0.02	0.0306	0.0248	0.0363	0.0118	0.0047	0.0199
Monetary policy shock (ϵ_m)	**IG**	0.001	0.02	0.0041	0.0034	0.0047	0.0033	0.0027	0.0040
Gov. spending shock (ϵ_a)	**IG**	0.001	0.02	0.0410	0.0340	0.0473	0.0298	0.0223	0.0370
Technology Persistence (ρ_a)	**IG**	0.50	0.20	0.9899	0.9829	0.9973	0.9948	0.9907	0.9991
Mark-up Persistence (ρ_{ms})	**IG**	0.50	0.20	0.9667	0.9505	0.9838	0.9716	0.9541	0.9881
Monetary policy shock Persistence (ρ_{ms})	**IG**	0.50	0.20	0.5032	0.4295	0.5829	0.6715	0.5803	0.7569
Gov. Spending Persistence (ρ_a)	**IG**	0.50	0.20	0.9103	0.8900	0.9307	0.9468	0.9244	0.9698
Calvo's parameter(ξ)	**B**	0.50	0.10	0.7011	0.6647	0.7378	0.3989	0.2828	0.5148
Labour's Frisch Elas.(ϕ)	**N**	2	0.75	3.7963	2.9980	4.5865	2.8489	1.8818	3.8011
Labour's scale(α)	**B**	0.50	0.10	0.8907	0.8350	0.9487	0.8787	0.8200	0.9384
Inattention parameters									
HH's General state ($\bar{m_h}$)	**B**	0.50	0.20				0.9925	0.9858	0.9993
HH's myopia interest (m_r)	**B**	0.50	0.20				0.3053	0.1712	0.4286
HH's myopia inflation (m_π)	**B**	0.50	0.20				0.0637	0.0074	0.1183
HH's myopia income (m_y)	**B**	0.50	0.20				0.4948	0.1720	0.8243
Firm's General state ($\bar{m}f$)	**B**	0.50	0.20				0.5542	0.2642	0.8841
Firm's myopia interest (m_r^f)	**B**	0.50	0.20				0.4942	0.1674	0.8226
Firm's myopia inflation (m_π^f)	**B**	0.50	0.20				0.5200	0.1970	0.8492
Firm's myopia marginal cost (m_x^f)	**B**	0.50	0.20				0.5243	0.2168	0.8128

Table 1: Estimation results - Parameter

3.2 Estimation results

Estimated results show that there is a significant difference in the general myopia levels about the future state between households and firms. While households' attentiveness level to the future state is relatively high, which is close to 1 or to the fully rational expectation, firms pay less attention to the future state. However, households have a smaller attentiveness level to the exogenous market's variables compared to firms.

Smets and Wouters (2003) and Smets and Wouters (2007) have shown that rational models with a rich set of frictions and a general stochastic structure can explain the data relatively well. However, these models require an implausibly high level of price and wage stickiness and exogenous shocks to explain the observed persistence in the data.[3] My estimated results show that the boundedly rational expectation reduces the scale of structural price-stickiness friction, ξ, and the magnitude of estimated shocks, which, most importantly, improves the marginal log likelihood relative to the RE model.

[3] Smets and Wouters (2007) resolve this problem by introducing Kimball rather than Dixit-stiglitz preferences. However, for Kimball preference to have a significant impact requires a huge super-price elasticity which is inconsistent with empirical evidence, Deak at all (2020). Hence, BR is an alternative approach to explain the persistence in observed data.

	RE (1)	BR (2)
Log Data Density	3812.98	3838.38

Table 2: Log data density, based on the Modified Harmonic Mean Estimator (Full sample 1959 to 2017 with Fed interest rate as nominal interest rate). The preference parameter, β, is calibrated at a fixed level of 0.9995. The usual procedure before was that we calibrated the mean of inflation and nominal interest rate over the data sample then we calculated the preference parameter, β, accordingly.

Based on the marginal log likelihood, the estimated model under BR expectation outperforms the rational expectation model at fitting with the data. The Bayesian odds is equal to $e^{(3838.38-3812.98)}$, the model odds are 0 for the RE model and 1 for the BR model. Hence, the RE model is firmly rejected by the data in favour of the BR model. The inattentive agents' misperceived representations of the economy in this case suggests a higher persistence current macroeconomic outcomes, which delivers a higher volatility, i.e. nominal interest rate and inflation to offset this misperceived behaviour from the public, compared to the fully rational case. Therefore, the behavioual model does a better job at matching with the persistences of the revised data. In addition, the important point here is that this behavioural framework seeks to study optimal central bank policy with behavioural agents. The inattention framework is a good candidate to approximate human beings' decisions. It seeks to be constructively skeptical of the rational expectations.

3.3 Second Moment Comparisons with Data

Sims (2003) address some concerns with Bayesian model comparison and the importance of seeking for the alternative criterion. In particular, Sims (2003) argues when using a single evidence in favor of a particular characteristic of the model while ignoring other factors can lead to disparate inference. In other words, the Bayesian model comparison is criticized on the basis of the argument that the models considered are too sparse. In such cases, posterior odds may lead to extreme outcomes. In addition, it is well known characteristic of the Bayesian approach when the estimated results are sensitive to the prior distributions. To further evaluate the absolute performance of one particular model against data, in this section we compare the models' implied characteristics (covariances and autocorrelations) with those of the actual data.

Table 3 presents some selected second moments implied by the above estimations and

Model	Output growth	Inflation	Interest rate	Worked hours
		Standard Deviation		
Data	0.0087	0.0058	0.0091	0.0360
Model RE	0.0089	0.0049	0.0070	0.0258
Model BR	0.0081	0.0063	0.0095	0.0268
		Cross-correlation with Output growth		
Data	1	-0.21	-0.09	0.16
Model RE	1.00	0.0508	0.0679	0.0967
Model BR	1.00	-0.0011	0.0370	0.0763
		Autocorrelations (Order=1)		
Data	0.30	0.88	0.97	0.98
Model RE	-0.0135	0.7500	0.9445	0.9521
Model BR	-0.0074	0.8843	0.9721	0.9675
		Autocorrelations (Order=4)		
Data	0.0479	0.7962	0.8293	0.8640
Model RE	-0.0161	0.4556	0.7629	0.8311
Model BR	-0.0105	0.6866	0.8627	0.8782
		Autocorrelations (Order=6)		
Data	0.0063	0.6995	0.7216	0.7702
Model RE	-0.0127	0.3807	0.6600	0.7632
Model BR	-0.0095	0.6174	0.7979	0.8246
		Autocorrelations (Order=8)		
Data	-0.0728	0.6311	0.6350	0.6770
Model RE	-0.0109	0.3305	0.5719	0.7026
Model BR	-0.0087	0.5695	0.7399	0.7750

Table 3: Selected Second Moments of the Model Variants

compares with those in the actual data. In particular, we compute these model-implied statistics by solving the models at the posterior means obtained from estimation. The results of the model's second moments are compared with the second moments in the actual data to evaluate the models' empirical performance. Specifically, the BR model performs relatively better than the RE model in matching actual data's statistics, such as standard deviation, and cross-correlation with output, but both models perform relatively poorly in some dimensions.

To further illustrate how the estimated models capture the data statistics and persistence in particular, we now plot the autocorrelations up to order 10 of the actual data and those of the endogenous variables generated by the model variants in Figure (1). Overall, both

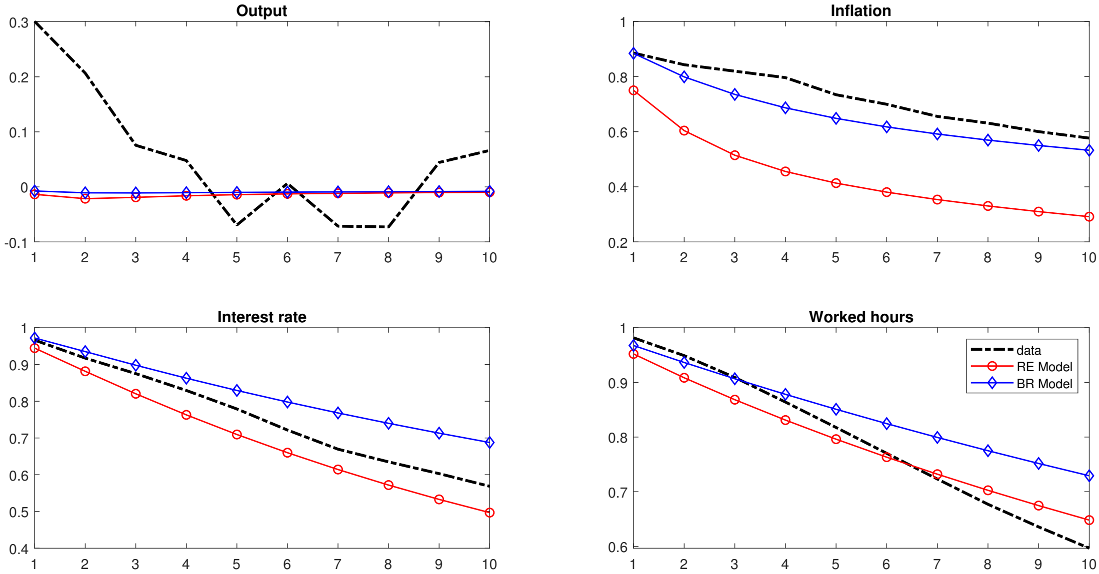

Figure 1: Autocorrelations of Observables in the Actual Data and in the Estimated Models. Sample period from 1958Q1 to 2017Q2 (Estimated sample)

models perform poorly in matching with actual output growth autocorrelations below order 4. However, the behavioural model does a better job at matching the persistence of actual data, especially regarding the inflation, nominal interest rate and worked hours series.

4 The Optimized ZLB Mandate

We first specify the central bank's simple nominal interest rate rule as follows:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\left(\theta_\theta \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t^c}{Y^c}\right) + \theta_{dy} \log\left(\frac{Y_t^c}{Y_{t-1}^c}\right)\right) \tag{62}$$

which for the purposes of computing the optimized rule we re-parameterize as:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \left(\alpha_\theta \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t^c}{Y^c}\right) + \alpha_{dy} \log\left(\frac{Y_t^c}{Y_{t-1}^c}\right)\right) \tag{63}$$

Following Deak et al. (2020), we now impose the ZLB with the following delegation game:

Stage 3: The CB Mandate Given a steady state inflation rate target, Π, the Central Bank (CB) receives a mandate to implement the rule (62) and to maximize with respect to ρ a modified welfare criterion[4]

$$\begin{aligned} V_t^{mod} &\equiv \mathbb{E}_t\left[\sum_{\tau=0}^{\infty} \beta^\tau \left(U_{t+\tau} - w_r\left(R_{n,t+\tau} - R_n\right)^2\right)\right] \\ &= \left(U_t - w_r\left(R_{n,t} - R_n\right)^2\right) + \beta \mathbb{E}_t\left[V_{t+1}^{mod}\right] \end{aligned} \tag{66}$$

One can think of this as a mandate with a penalty function $P = w_r\left(R_{n,t} - R_n\right)^2$, penalizing the variance of the nominal interest rate with weight w_r,.

[4] Following Woodford (2003) and Levine et al. (2008), we impose an approximate form of the ZLB constraint which requires that the average value of $R_{n,t}^2$ is not more than K ($K = 1 + k^2 > 1$) times the square of the average value of $R_{n,t}$.

$$\mathbb{E}_t\left[(1-\beta)\sum_{\tau=0}^{\infty} \beta^\tau R_{n,t+\tau}^2\right] \leq K\left[\mathbb{E}_t\left[(1-\beta)\sum_{\tau=0}^{\infty} \beta^\tau R_{n,t+\tau}\right]^2\right] \tag{64}$$

Using discounted averages we have

$$\bar{R_n} \geq k\sqrt{\overline{(R_{n,t} - \bar{R_n})}^2} = k\sqrt{\bar{R_{n,t}}^2 - \bar{R_n}^2} \tag{65}$$

From the initial optimization of consumer's expected utility we now have an additional constraint in form of equation (64), I add a term $w_r(\bar{R_{n,t}}^2 - K\bar{R_n}^2)$ to the Lagrangian to incorporate this constraint with w_r is the Lagangian multiplier which is strictly larger than zero. Then the new optimization problem has the same constraints as the original problem but its objective function is now included an additional term in the form of $w_r(R_{n,t} - \bar{R_n})^2$. In addition, more in line with CB practice, mandates could also take the form of a quadratic loss function with explicit targets for output, inflation and the nominal interest rates variances.

Mandate results in a probability of hitting the ZLB

$$p = p(\Pi, \rho^*(\Pi, w_r)) \tag{67}$$

where $\rho^*(\Pi, w_r)$ is the optimized form of the rule given the steady state target Π and the weight on the interest rate volatility, w_r.

Stage 2: Choice of the Steady State Inflation Rate Π Given a target low probability $\bar{p}$ and given w_r, $\Pi = \Pi^*$ is chosen so satisfy

$$p(R_{n,t} \leq 1) \equiv p(\Pi^*, \rho^*(\Pi^*, w_r)) \leq \bar{p} \tag{68}$$

This then achieves the ZLB constraint

$$R_{n,t} \geq 1 \text{ with high probability } 1 - \bar{p} \tag{69}$$

where $R_{n,t}$ is the nominal interest rate.

Stage 1: Design of the Mandate The policymaker first chooses a per period probability $\bar{p}$ of the nominal interest rate hitting the ZLB (which defines the tightness of the ZLB constraint). Then it maximizes the actual household intertemporal welfare

$$V_t = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} U_{t+\tau} \right] = U_t + \beta \mathbb{E}_t \left[V_{t+1} \right] \tag{70}$$

with respect to w_r.

This three-stage delegation game defines an equilibrium in choice variables w_r^*, ρ^* and Π^* that maximizes the true household welfare subject to the ZLB constraint (69).

4.1 The Optimized Simple Rule

We first check for the optimized simple rule without the ZLB consideration. Virtually, results are presented in figures (2), (3). Overall, the probability of the nominal interest hitting the ZLB is significantly different across models. The first subplot of figures (2) and (3) shows the relationship between the ZLB probability for each value of the gross steady state inflation rate. In particular, the probability of hitting the ZLB is a decreasing function in the level of the inflation target. Second subplot indicates that the nominal interest rates' standard deviations is increasing in the levels of steady state inflation. Therefore, increasing the inflation target has two opposite effects on the probability of hitting the ZLB: the first is on the first moment by shifting the density function to the right reducing the probability of hitting ZLB; the second effect is on the second moment making the shape of the density function more fat-tailed. The third and forth subplots illustrate that welfare in term of consumption is always a decreasing function in the levels of steady state inflation for the NK models.

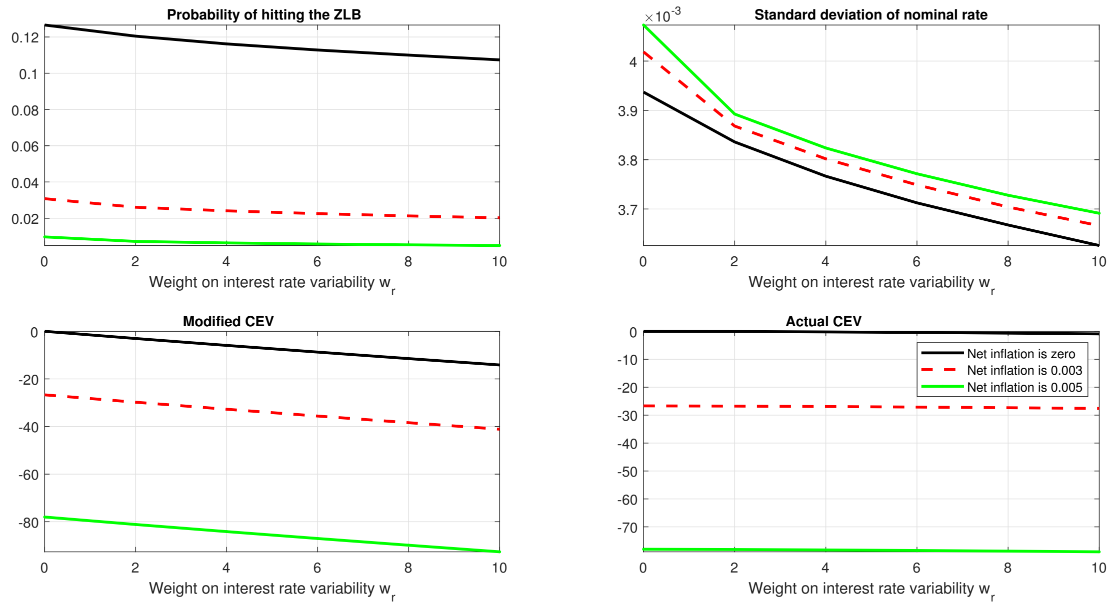

Figure 2: RE model stage 3

First, when we impose the penalty term on the nominal interest volatility, the optimized simple rules converge to a price-level targeting rule in both models. However, under the BR model, the central bank reacts less aggressive to their macroeconomic targets. As it is well pointed out in Gabaix (2020) that a simple nominal interest rate rule under BR has a larger determinacy region compared to an identical rule under the rational expectation model, which allows the central bank to operate with a lower weight on inflation in their optimized rule. Second, under the BR model, the nominal interest rate is more volatile, which can be explained by several factors, such as the different magnitude of the estimated shocks in each model. However, it is clear that under the BR the central bank has their interest

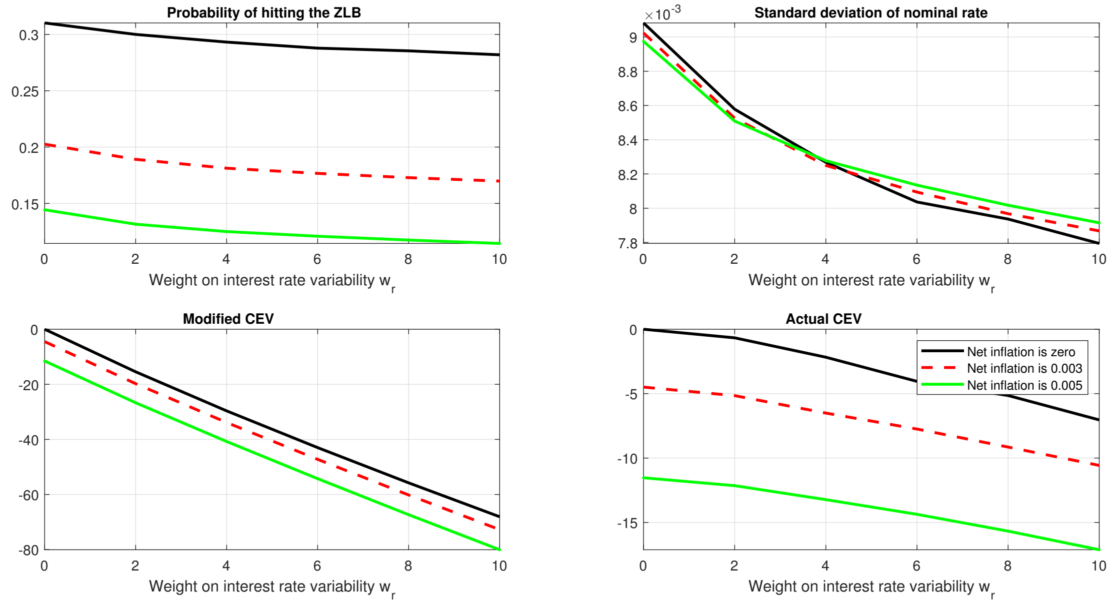

Figure 3: BR model stage 3

rate reacting more aggressive in order to, first offset the shocks, and second compensate for the inattention level on the policy rate by the public compared to the fully rational model. Finally, the probability of the nominal interest rate hitting the ZLB is significantly different across models. In particular, under BR this probability is significantly higher, which results directly from the higher volatility of the nominal interest rate aforementioned above.

Table (4) provides additional results on the optimized simple rules with ZLB. First, if the weight attached on the penalty term is higher, the optimized simple rule converges to a price level target rule. It is well known that the price level target rule would allow the central bank to have a higher inflation volatility because the inflation can be under/over shoot this period and then compensated by an over/under shoot for the next periods. Hence, the feedback parameter on inflation of the optimal rule is decreasing along the convergence to the price lvel target rule. Second, standard deviation on the rate is decreasing with the weight w_r because it is the direct effect of the penalty term, which comes along with the decreasing probability of the rate hitting the ZLB. Finally, given the same level of w_r and steady state inflation, BR model produces a significantly higher welfare loss compared to the case of RE model. In addition, the probability of the rate hitting the ZLB is much

	(A) RE model						
w_r	ρ_r	α_π	α_y	α_{dy}	SD(R_n)	Pr(ZLB)	$cev = 100\frac{\Omega(0)-\Omega(w_r)}{CE}(\%)$
0	0.99	6.89	0.00	0.37	0.0039	0.1266	0
2	1.00	4.10	0.00	0.22	0.0038	0.1205	-0.00007
4	1.00	3.01	0.00	0.16	0.0038	0.1162	-0.00023
6	1.00	2.43	0.00	0.13	0.0037	0.1128	-0.00044
8	1.00	2.07	0.00	0.10	0.0037	0.1100	-0.00069
10	1.00	1.79	0.00	0.09	0.0036	0.1074	-0.00097
	(B) BR model						
w_r	ρ_r	α_π	α_y	α_{dy}	SD(R_n)	Pr(ZLB)	$cev = 100\frac{\Omega(0)-\Omega(w_r)}{CE}(\%)$
0	0.28	3.71	0.026	0.29	0.009	0.31	0
2	0.855	1.108	0.007	0.086	0.0086	0.30	-0.00067
4	0.956	0.645	0.004	0.055	0.0083	0.29	-0.00217
6	0.996	0.47	0.003	0.043	0.0083	0.288	-0.00405
8	0.996	0.38	0.002	0.041	0.008	0.285	-0.00514
10	1.000	0.31	0.00	0.03	0.0078	0.282	-0.00704

Table 4: Optimized Rule with a ZLB

higher under BR. Therefore, when we consider the ZLB of a given probability, the higher welfare loss of BR compared to RE comes from two sources, first the model volatility, second the higher welfare cost of the ZLB.

4.2 The Optimized Simple Rule with a ZLB Constraint

In this section, we impose the ZLB constraint (69) where the optimal inflation target is chosen to maximize the welfare at each w_r as explained in the previous section.

In order to examine the model's behaviour under the binding ZLB constraint we set the value of $\bar{p} = 0.01$ quarterly. Figures (4) and (5) represent the behaviours of some variables under the binding ZLB constraint for the RE and BR models, respectively. The first plot of each figure shows the minimum values of steady state inflation rate, Π^*, which satisfies equation (68) in stage 2 with equality. As shown in fourth subplot of figures (2) and (3), the level of welfare is a decreasing function of the inflation target values. Therefore, the central bank will set the lowest inflation target satisfying the ZLB.

Overall, the two models replicate equilibrium behaviours as presented in Deak et al. (2020). However, there is a significant difference of the equilibrium points between models. In particular, under the rational expectation model, the optimal weight imposed on the penalty term of ZLB mandate is relatively high, $w_r^* = 64$ compared to $w_r^* = 21$ under BR model. One interesting point to be mentioned here is that under the BR model the volatility of nominal interest first decreases in the level of steady state inflation for small

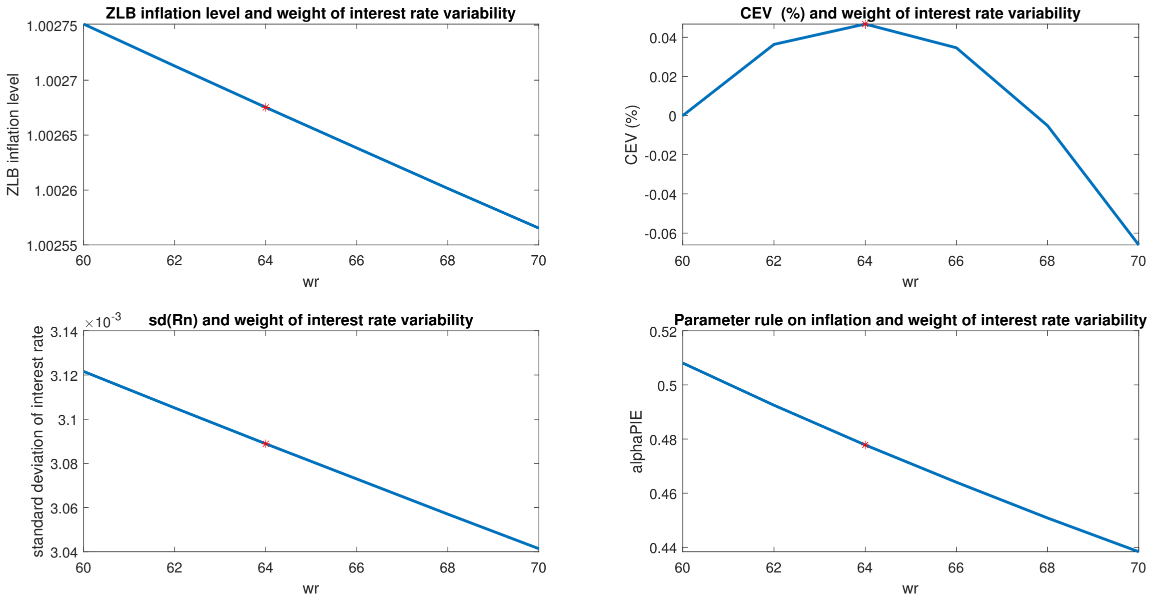

Figure 4: RE model optimized simple rule imposed ZLB at $\bar{p} = 0.01$

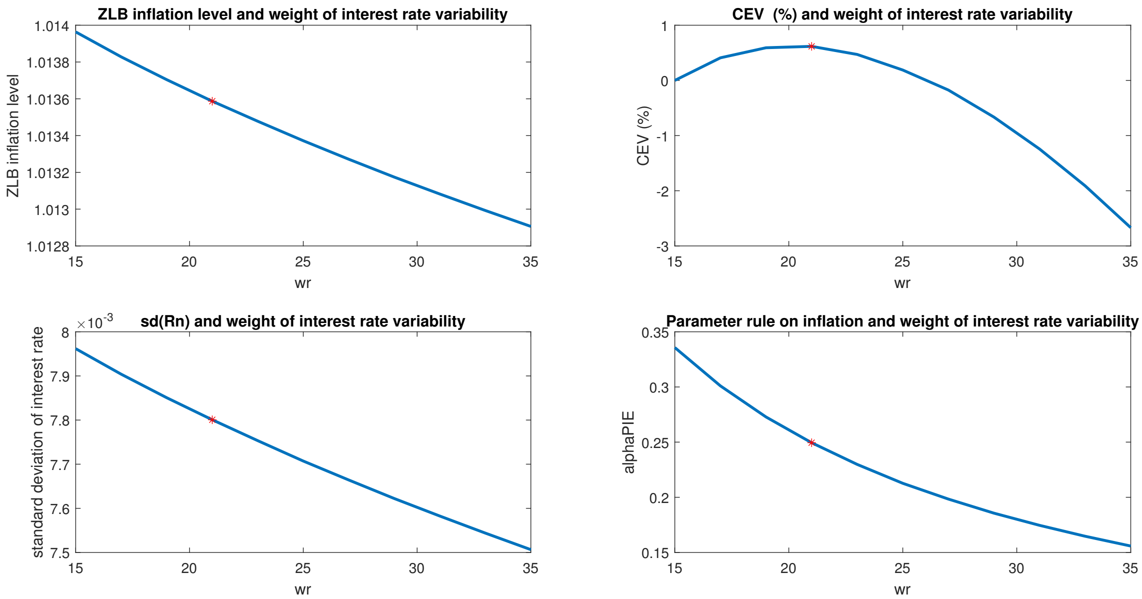

Figure 5: BR model optimized simple rule imposed ZLB at $\bar{p} = 0.01$

values of w_r; it then, however, increases in the steady state inflation when w_r gets larger, i.e. when $w_r > 4$ as shown in subplot 2 of the figure (3). In principle, as shown from the rational expectation model, the direct effect of steady state inflation levels on the volatility of nominal interest rate is positive. Higher inflation leads to a higher model's volatility, overall, which results in higher nominal interest rate's variations. However, under small levels of weight w_r, the BR model produces an optimized simple rule with a low persistence in the nominal interest rate inertia parameter as shown in table (**??**). This overall mitigates the impact of a higher steady state inflation on the nominal interest rate's variations. Hence, the optimal weight w_r under BR model is significantly smaller compared the that under the rational expectation model. In addition, the central bank has tendency to be less aggressive in conducting its monetary policy in term of stabilizing the price, which leads to a fall in feedback rule parameter on inflation under both model. Finally, the equilibrium is represented by the red dotted point. The exact equilibrium is as follows.

	(1) Welfare comparison under Rational Expectation model									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act welfare	CEV (%)	p_zlb	w_r^*	MPS
OSR without ZLB ($\Pi = 1.0$)	0.99	6.89	0.00	0.37	1.0	-2565.491	0	0.13	0	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.48	0.00	0.012	1.0027	-2566.179	-0.0360	0.01	64	0.0
	(1) Welfare comparison under BR model									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π	Act welfare	CEV (%)	p_zlb	w_r^*	MPS
OSR without ZLB ($\Pi = 1.0$)	0.28	3.71	0.026	0.29	1.0	-2949.072	0	0.31	0	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.25	0.00	0.018	1.0136	-2950.980	-0.100	0.01	21	0.0

Table 5: Welfare cost of the ZLB under different expectation formations

In the BR model, the welfare cost of the ZLB is significantly higher. While Gabaix (2020) studies welfare of the ZLB relative to output level at the exact ZLB episodes and given a zero net-inflation trend, we instead consider the ZLB in term of the steady state inflation which is the main source of welfare cost in standard NK model. In this framework, a higher volatility of nominal interest rate is translated into a longer lasting ZLB episode, which in turn results in a higher steady state inflation. As shown in table (5) optimal inflation under BR model is close to 5 % annually compared to 1 % under RE model. Therefore, a welfare cost to overcome the ZLB under BR model is significantly higher than that under the RE model. This is a major policy implication of relaxing RE for the behavioural model.

Consider a simple case, for instance, a positive mark-up (cost-push) shock. The reaction of myopic price-setting firm is to increase the price by more than a rational price-setting firm would. This effect will transmit to inflation. As a result, the outcomes of both inflation and output under BR will be worse than in the RE, and consequently, the associated welfare will decrease. In addition, with a positive inflation trend this effect would be amplified even more under BR setup. Which in turn requires a significantly higher monetary policy expansion to offset this negative effect on output and price stability under the BR case. Hence, a large nominal interest rate's variance leads to a higher probability of the rate hitting the ZLB under BR. However, if we consider a case without the ZLB on nominal interest rate, BR agents form their expectations based on misperceived representations of the economy, which produces smoother current macroeconomic outcomes, i.e. output and inflation variations would be smaller because of the myopia parameters. Consequently, BR delivers moderate welfare losses compared to the RE case. Therefore, in this framework of ZLB when the nominal interest rate's variance plays the most crucial role in determining the welfare level in the economy, the BR's smoother macroeconomic outcomes is trivial.

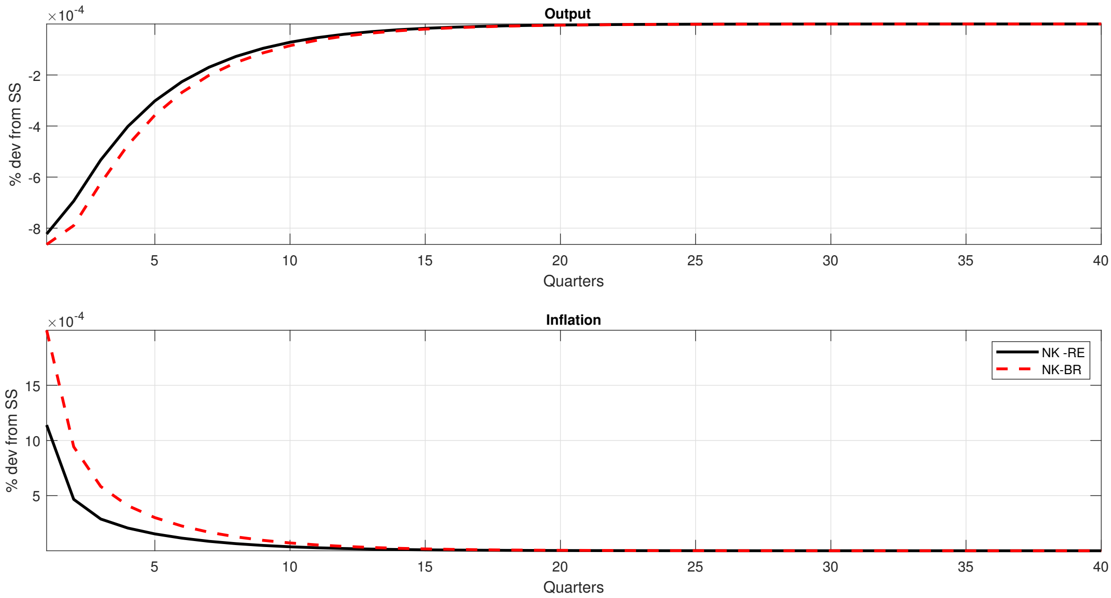

Figure 6: Output and inflation reactions to a positive mark-up shock

In addition, it is worth pointing out that, with or without the ZLB consideration, the feedback parameter on inflation of the optimized simple rule of the form (62) shows in the long-run that the general Taylor principle that the nominal interest rate reacts more than one to current inflation.

4.3 The Welfare Cost of Suboptimal Policy

In this section we quantify the cost of model j implementing the mandate designed for model i $i \neq j$. For example, if the BR model uses the optimized mandate of the rational expectation model which contains the optimized simple rule and the optimal steady state inflation, $[\rho^*_{RE,rule}, \Pi^*_{RE}]$, then welfare gain is 0.0917 % of consumption relative to optimal mandate designed for BR model itself. Similarly, if the rational expectation model deploys the optimal mandate designed for the BR model, then the welfare gain will be -0.791 % of consumption compared to the case when RE model uses the optimal mandate designed for itself, as shown in table (6).

	Rational Expectation (RE) Model	Rational Inattention (BR) model
RE Mandate Equilibrium	-2566.179 (CEV = 0 %)	-2949.229 (CEV = 0.0917 %)
BR Mandate Equilibrium	-2581.286 (CEV = -0.791 %)	-2950.980 (CEV = 0 %)

Table 6: Welfare cost of suboptimal mandate

It is worth pointing out that the optimal mandate designed for the RE model is actually **more robustly** optimal across models in this framework because the optimal inflation of the RE's optimal mandate is very close to zero, which generates a modest welfare cost compared to the optimal annual inflation for the BR model of almost 5 %. As mentioned in the previous section, the ZLB framework in this paper is the main determinant of welfare. In particular, under BR model the ZLB is significantly costlier in term of welfare, which is translated into a much higher level of steady state inflation. Notice we do not directly compare the welfare between RE and BR, but we are comparing the welfare cost between transition stages under each expectation formation. Therefore, the welfare differences of the comparison here represents the misspecifications of these model. However, the welfare criteria here clearly shows that under the ZLB consideration the rule designed for the RE model produces the best outcome for the society when faced with model uncertainty.

5 Conclusions

In this paper, we extend the New Keynesian model to account for agents inattentiveness to macroeconomic variables by relaxing the rationality assumption in favor of bounded rationality (Gabaix, 2020), whereby agents are assumed to be partially myopic and unable to anticipate macroeconomic developments perfectly. In general, we assume that agents form beliefs over the future infinite time horizon of aggregate states and prices which are exogenous to their decisions. we then directly apply agents' inattentiveness on these exogenous market aggregate states and prices into their infinite-time-horizon decision rules.

A main contributions the paper is to contrast the effect of the ZLB constraint on an optimized Taylor-type nominal interest rate for the RE and behavioural frameworks. A main result is to show that the ZLB welfare cost under BR significantly dominates the other welfare costs of the business cycle. A further important finding is to show that there is a significant disparity in optimized monetary mandates produced by the different models of expectations by agents. In a likelihood race, the estimated behavioural model by far produces the best fit to the data. However robustness considerations suggest that faced with model uncertainty across the two models it is actually best to design policy using the RE variant of this estimated model.

In this paper we have adopted only one possible model of BR, whereas there is now a substantial a literature on behavioural macroeconomics, for NK models in particular, with different boundedly rational expectations assumptions (see Eusepi and Preston (2018), Branch and McGough (2018) and Jump and Levine (2018) for some recent surveys). Therefore, a question for monetary policy makers is how should they deploy this potential pool of bounded rationality expectations models to design optimal monetary policy in the face of model uncertainty? Future work, that follows naturally from the study in this paper, will use the optimal pooling methodology set out in Deak et al. (2019) to address this question by studying *robust* optimal monetary policy. The study will first design an optimal pooling weights across alternative models which will include those with Euler Learning, Anticipated Utility, BR and RE. Then a common optimized robust rule will be computed, using the technique adopted in this paper, to optimise a weighted average of the household inter-temporal welfare evaluated for each model. This avenue of research will then be a disciplined modelling response to Sargent (1999) who has famously commented that abandoning rational expectations can lead a modeller into a "*wilderness of bounded rationality*".

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Appendices

A The Rational Expectations Model

A.1 Households

Household j chooses between work and leisure and therefore how much labour they supply. Let $C_t(j)$ and $H_t(j)$ denote consumption and labour supply, respectively. The single-period utility is given by

$$U_t(j) = U(C_t(j), H_t(j)) = \log(C_t(j)) - \kappa \frac{H_t(j)^{1+\phi}}{1+\phi} \tag{71}$$

Note: we should introduce habit formation. Instead of substituting the Euler equation into (42) we should just keep $\mathbb{E}_t^* C_{t+i} = \mathbb{E}_t^* C_{t+1}$ in the equation as long as possible. We can factor it out of the summation and substitute in the RHS of the Euler at the very end only. If we use external habit, then the resulting Euler equation can easily be solved for $\mathbb{E}_t^* C_{t+i}$. With internal habit this may not be the case anymore.

In a stochastic environment, the value function of the representative household at time t is given by

$$V_t(j) = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U_{t+s}(j) \right] \tag{72}$$

The household's problem at time t is to choose paths for consumption $\{C_t(j)\}$, labour supply $\{H_t(j)\}$ and holdings of financial assets $\{B_t(j)\}$ to maximize $V_t(j)$ given by (72) given its budget constraint in period t

$$B_t(j) = R_t B_{t-1}(j) + W_t H_t(j) + \Gamma_t - C_t(j) - T_t \tag{73}$$

where $B_t(j)$ is holdings of financial assets at the end of period t, W_t is the real wage rate, R_t is the interest rate paid on assets held at the beginning of period t, Γ_t are profits from wholesale and retail firms owned by households and T_t denote taxes. W_t, R_t, Γ_t and T_t are all exogenous to household j.

To solve the household problem we form a Lagrangian

$$\mathcal{L} = \mathbb{E}_t \Bigg[\sum_{s=0}^{\infty} \beta^s \Bigg\{ U_{t+s}(j) \\ + \lambda_{t+s}(j) \left[R_{t+s} B_{t+s-1}(j) + W_{t+s} H_{t+s}(j) + \Gamma_{t+s} - C_{t+s}(j) - T_{t+s} - B_{t+s}(j) \right] \Bigg\} \Bigg] \tag{74}$$

The first-order conditions with respect to $\{C_{t+s}(j)\}$, $\{B_{t+s}(j)\}$ and $\{H_{t+s}(j)\}$ are

$$\begin{aligned}
\{C_{t+s}(j)\}: &\quad \mathbb{E}_t \beta^s U_{C,t+s}(j) + \beta^s \lambda_{t+s}(j) = 0 \\
\{B_{t+s}(j)\}: &\quad \mathbb{E}_t \left[\beta^{s+1} \lambda_{t+s+1}(j) R_{t+s+1} \right] - \beta^s \lambda_{t+s}(j) = 0 \\
\{H_{t+s}(j)\}: &\quad \mathbb{E}_t \left[\beta^s U_{H,t+s}(j) + \beta^s \lambda_{t+s}(j) W_{t+s} \right] = 0
\end{aligned}$$

Rearranging the first-order conditions we get:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1}(j) R_{t+1} \right] \tag{75}$$

$$W_t = -\frac{U_{H,t}(j)}{U_{C,t}(j)} \tag{76}$$

where

$$\Lambda_{t,t+1}(j) = \beta \frac{U_{C,t+1}(j)}{U_{C,t}(j)} \tag{77}$$

$$U_{C,t} = \frac{1}{C_t} \tag{78}$$

$$U_{H,t} = -\kappa H_t^{\phi} \tag{79}$$

is the real stochastic discount factor for household j over the interval $[t, t+1]$.

A.2 Firms in the Wholesale

Wholesale firms employ a Cobb-Douglas production function to produce a homogeneous output

$$Y_t^W = F(A_t, H_t) = A_t H_t^{\alpha} \tag{80}$$

where A_t is total factor productivity. Profit-maximizing demand for labour results in the first order condition

$$W_t = \frac{P_t^W}{P_t} F_{H,t} = \alpha \frac{P_t^W}{P_t} \frac{Y_t^W}{H_t} \tag{81}$$

A.3 Firms in the Retail Sector

The retail sector uses a homogeneous wholesale good to produce a basket of differentiated goods for aggregate consumption

$$C_t = \left(\int_0^1 C_t(m)^{(\zeta-1)/\zeta} dm \right)^{\zeta/(\zeta-1)} \tag{82}$$

where ζ is the elasticity of substitution. For each m, the consumer chooses $C_t(m)$ at a price $P_t(m)$ to maximize (82) given total expenditure $\int_0^1 P_t(m) C_t(m) dm$. This results in a set of consumption demand equations for each differentiated good m with price $P_t(m)$ of the form

$$C_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} C_t \Rightarrow Y_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} Y_t \tag{83}$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta} dm \right]^{\frac{1}{1-\zeta}}$. P_t is the aggregate price index. C_t and P_t are Dixit-Stigliz aggregates – see Dixit and Stiglitz (1977).

For each variety m the retail good is produced from wholesale production according to an iceberg technology

$$Y_t(m) = Y_t^W = A_t H_t(m)^\alpha \tag{84}$$

Following Calvo (1983), we now assume that there is a probability of $1 - \xi$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then it is held fixed.[5] For each retail producer m, given its real marginal cost MC_t, the objective is at time t to choose $\{P_t^0(m)\}$ to maximize discounted profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - P_{t+k} MC_{t+k} \right] \tag{85}$$

subject to (83). The solution to this is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - \frac{1}{(1-1/\zeta)} P_{t+k} MC_{t+k} \right] = 0 \tag{86}$$

which leads to

$$\frac{P_t^0(m)}{P_t} = \frac{1}{1-1/\zeta} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta-1} Y_{t+k}} \tag{87}$$

[5] Thus we can interpret $\frac{1}{1-\xi}$ as the average duration for which prices are left unchanged.

where k periods ahead inflation is defined by

$$\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t} = \frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}} \cdot \cdot \frac{P_{t+k}}{P_{t+k-1}} = \Pi_{t+1}\Pi_{t+2} \cdot \cdot \Pi_{t+k}$$

Note that $\Pi_{t,t+1} = \Pi_{t+1}$ and $\Pi_{t,t} = 1$.

Let us define

$$\begin{aligned} J_t &= \frac{1}{1-\frac{1}{\zeta}}\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\zeta} Y_{t+k} MC_{t+k} \\ &= \frac{1}{1-\frac{1}{\zeta}} Y_t MC_t + \xi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} J_{t+1} \quad (88) \\ JJ_t &= \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\zeta-1} Y_{t+k} \\ &= Y_t + \xi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} JJ_{t+1} \quad (89) \end{aligned}$$

Then (87) can be written as

$$\frac{P_t^0(m)}{P_t} = \frac{J_t}{JJ_t} \quad (90)$$

By the law of large numbers the evolution of the price index is given by

$$P_{t+1}^{1-\zeta} = \xi P_t^{1-\zeta} + (1-\xi)(P_{t+1}^0)^{1-\zeta} \quad (91)$$

which can be written as

$$1 = \xi \Pi_t^{\zeta-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{1-\zeta} \quad (92)$$

Price dispersion is defined as $\Delta_t = \int (P_t(m)/P_t)^{-\zeta}$. Assuming that the number of firms is large, we obtain the following dynamic relationship:

$$\begin{aligned} \Delta_t &= \xi \int_{not\ optimize} \left(\frac{P_{t-1}^0(m)}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{-\zeta} + (1-\xi)\int_{optimize}\left(\frac{P_t^0(m)}{P_t}\right)^{-\zeta_p} \\ &= \xi \Pi_t^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{P_t^0(m)}{P_t}\right)^{-\zeta} \\ &= \xi \Pi_t^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{-\zeta} \quad (93) \end{aligned}$$

A.4 Profits

Total profits from retail and wholesale firms, Γ_t, are remitted to households. This is given in real terms by

$$\Gamma_t = \underbrace{Y_t - \frac{P_t^W}{P_t} Y_t^W}_{\text{retail}} + \underbrace{\frac{P_t^W}{P_t} Y_t^W - W_t H_t}_{\text{Wholesale}} = Y_t - \alpha \frac{P_t^W}{P_t} Y_t^W \tag{94}$$

using the first-order condition (81).

A.5 Closing the Model

The model is closed with a resource constraint

$$Y_t = C_t + G_t \tag{95}$$

and the government's budget constraint

$$G_t = T_t \tag{96}$$

Market clearing in the goods market requires

$$\int_0^1 Y_t(m) dm = \int_0^1 \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} Y_t dm = Y_t \Delta_t \tag{97}$$

using (83). Hence in a symmetric equilibrium

$$Y_t^W = Y_t \Delta_t \tag{98}$$

A monetary policy rule for the nominal interest rate is given by the following Taylor-type rule

$$\log \left(\frac{R_{n,t}}{R_n} \right) = \rho_r \log \left(\frac{R_{n,t-1}}{R_n} \right) + (1 - \rho_r) \left(\theta_\theta \log \left(\frac{\Pi_t}{\Pi} \right) + \theta_y \log \left(\frac{Y_t}{Y} \right) \right) + \log MPS_t, \tag{99}$$

where MPS_t is a monetary policy shock. The ex ante nominal gross interest rate $R_{n,t}$ set at time t and the ex post real interest rate, R_t are related by the Fischer equation

$$R_t = \frac{R_{n,t-1}}{\Pi_t} \tag{100}$$

Exogenous processes evolve according to:

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{101}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{102}$$

$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{103}$$

A.6 Equilibrium

A symmetric equilibrium is determined by the following equations:

$$U_t = \log(C_t) - \kappa \frac{H_t^{1+\phi}}{1+\phi} \tag{104}$$

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U_{t+s} \right] = U_t + \beta \mathbb{E}_t V_{t+1} \tag{105}$$

$$U_{C,t} = \frac{1}{C_t} \tag{106}$$

$$U_{H,t} = -\kappa H_t^{\phi} \tag{107}$$

$$\Lambda_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}} \tag{108}$$

$$R_t = \frac{R_{n,t-1}}{\Pi_t} \tag{109}$$

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1} \right] \tag{110}$$

$$W_t = -\frac{U_{H,t}}{U_{C,t}} \tag{111}$$

$$Y_t^W = A_t H_t^{\alpha} \tag{112}$$

$$W_t = \alpha \frac{P_t^W}{P_t} \frac{Y_t^W}{H_t} \tag{113}$$

$$MC_t = \frac{P_t^W}{P_t} \tag{114}$$

$$J_t = \frac{1}{1-\frac{1}{\zeta}} Y_t MC_t MS_t + \xi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} J_{t+1} \tag{115}$$

$$JJ_t = Y_t + \xi \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} JJ_{t+1} \tag{116}$$

$$1 = \xi \Pi_t^{\zeta-1} + (1-\xi) \left(\frac{J_t}{JJ_t} \right)^{1-\zeta} \tag{117}$$

$$Y_t = \frac{Y_t^W}{\Delta_t} \tag{118}$$

$$\Delta_t = \xi \Pi_t^{\zeta} \Delta_{t-1} + (1-\xi) \left(\frac{J_t}{JJ_t} \right)^{-\zeta} \tag{119}$$

$$Y_t = C_t + G_t \tag{120}$$

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\left(\theta_\theta \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right)\right) + \log MPS_t \tag{121}$$

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{122}$$

$$\log MS_t - \log MS = \rho_{MS}(\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \tag{123}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{124}$$

$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{125}$$

where we have introduced a mark-up shock MS_t.

A.7 Stationary equilibrium

Labour-augmenting technical progress parameter is decomposed into a cyclical component, A_t^c, and a deterministic trend $\bar{A}_t$:

$$A_t = \bar{A}_t A_t^c$$

$$\bar{A}_t = (1+g)\bar{A}_{t-1}$$

Rewrite the equilibrium conditions as

$$U_t - \log(\bar{A}_t) = \log(C_t/\bar{A}_t) - \kappa \frac{H_t^{1+\phi}}{1+\phi} \tag{126}$$

$$V_t = U_t + \beta \mathbb{E}_t V_{t+1} \tag{127}$$

$$\bar{A}_t U_{C,t} = \frac{1}{C_t/\bar{A}_t} \tag{128}$$

$$U_{H,t} = -\kappa H_t^\phi \tag{129}$$

$$\Lambda_{t,t+1} = \frac{\beta}{\bar{A}_{t+1}/\bar{A}_t} \frac{\bar{A}_{t+1} U_{C,t+1}}{\bar{A}_t U_{C,t}} \tag{130}$$

$$R_t = \frac{R_{n,t-1}}{\Pi_t} \tag{131}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}\right] \tag{132}$$

$$\frac{W_t}{\bar{A}_t} = -\frac{U_{H,t}}{\bar{A}_t U_{C,t}} \tag{133}$$

$$\frac{Y_t^W}{\bar{A}_t} = \frac{A_t}{\bar{A}_t} H_t^\alpha \tag{134}$$

$$\frac{W_t}{\bar{A}_t} = \alpha \frac{P_t^W}{P_t} \frac{Y_t^W/\bar{A}_t}{H_t} \tag{135}$$

$$MC_t = \frac{P_t^W}{P_t} \tag{136}$$

$$\frac{J_t}{\bar{A}_t} = \frac{1}{1-\frac{1}{\zeta}} \frac{Y_t}{\bar{A}_t} MC_t MS_t + \xi \mathbb{E}_t \frac{\bar{A}_{t+1}}{\bar{A}_t} \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} \frac{J_{t+1}}{\bar{A}_{t+1}} \tag{137}$$

$$\frac{JJ_t}{\bar{A}_t} = \frac{Y_t}{\bar{A}_t} + \xi \mathbb{E}_t \frac{\bar{A}_{t+1}}{\bar{A}_t} \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} \frac{JJ_{t+1}}{\bar{A}_{t+1}} \tag{138}$$

$$1 = \xi \Pi_t^{\zeta-1} + (1-\xi) \left(\frac{J_t/\bar{A}_t}{JJ_t/\bar{A}_t} \right)^{1-\zeta} \tag{139}$$

$$\frac{Y_t}{\bar{A}_t} = \frac{Y_t^W/\bar{A}_t}{\Delta_t} \tag{140}$$

$$\Delta_t = \xi \Pi_t^{\zeta} \Delta_{t-1} + (1-\xi) \left(\frac{J_t/\bar{A}_t}{JJ_t/\bar{A}_t} \right)^{-\zeta} \tag{141}$$

$$\frac{Y_t}{\bar{A}_t} = \frac{C_t}{\bar{A}_t} + \frac{G_t}{\bar{A}_t} \tag{142}$$

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\ &\quad + (1-\rho_r)\left(\theta_\theta \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right)\right) + \log MPS_t \end{aligned} \tag{143}$$

$$\log A_t - \log A = \rho_A (\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{144}$$

$$\log MS_t - \log MS = \rho_{MS} (\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \tag{145}$$

$$\log MPS_t - \log MPS = \rho_{MPS} (\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{146}$$

$$\log G_t - \log G = \rho_G (\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{147}$$

Use change of variables to arrive to the following equilibrium conditions:[6]

$$U_t^c = \log(C_t^c) - \kappa \frac{H_t^{1+\phi}}{1+\phi} \tag{148}$$

$$V_t^c = U_t^c + \beta \mathbb{E}_t V_{t+1}^c \tag{149}$$

$$U_{C,t}^c = \frac{1}{C_t^c} \tag{150}$$

$$U_{H,t} = -\kappa H_t^{\phi} \tag{151}$$

$$\Lambda_{t,t+1} = \frac{\beta}{1+g} \frac{U_{C,t+1}^c}{U_{C,t}^c} \tag{152}$$

[6]The first equation is based on a hunch. Since the normalization of utility is additive, we cannot have a different discount factor. However, we cannot derive the first equation above from (105). We can derive it starting from the definition $V_t^c = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U_{t+s}^c\right]$.

$$R_t = \frac{R_{n,t-1}}{\Pi_t} \tag{153}$$

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}\right] \tag{154}$$

$$W_t^c = -\frac{U_{H,t}}{U_{C,t}^c} \tag{155}$$

$$Y_t^{W,c} = A_t^c H_t^\alpha \tag{156}$$

$$W_t^c = \alpha \frac{P_t^W}{P_t} \frac{Y_t^{W,c}}{H_t} \tag{157}$$

$$MC_t = \frac{P_t^W}{P_t} \tag{158}$$

$$J_t^c = \frac{1}{1-\frac{1}{\zeta}} Y_t^c MC_t MS_t + \xi(1+g)\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} J_{t+1}^c \tag{159}$$

$$JJ_t^c = Y_t^c + \xi(1+g)\mathbb{E}_t \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} JJ_{t+1}^c \tag{160}$$

$$1 = \xi \Pi_t^{\zeta-1} + (1-\xi)\left(\frac{J_t^c}{JJ_t^c}\right)^{1-\zeta} \tag{161}$$

$$Y_t^c = \frac{Y_t^{W,c}}{\Delta_t} \tag{162}$$

$$\Delta_t = \xi \Pi_t^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{J_t^c}{JJ_t^c}\right)^{-\zeta} \tag{163}$$

$$Y_t^c = C_t^c + G_t^c \tag{164}$$

$$\begin{aligned}\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\ &\quad + (1-\rho_r)\left(\theta_\theta \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t^c}{Y^c}\right)\right) + \log MPS_t\end{aligned} \tag{165}$$

$$\log A_t^c - \log A^c = \rho_A(\log A_{t-1}^c - \log A^c) + \epsilon_{A,t} \tag{166}$$

$$\log MS_t - \log MS = \rho_{MS}(\log MS_{t-1} - \log MS) + \epsilon_{MS,t} \tag{167}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{168}$$

$$\log G_t^c - \log G^c = \rho_G(\log G_{t-1}^c - \log G^c) + \epsilon_{G,t} \tag{169}$$

This is a system of 22 equation in the following 22 “variables” (in order of appearance): V^c, U^c, C^c, H, Λ, R, W^c, U_H^c, U_C^c, $Y^{W,c}$, A^c, $\frac{P^W}{P}$, J^c, Y^c, MC, MS, Π, JJ^c, Δ, G^c, R_n, MPS.

A.8 Steady State

The exogenous variables have steady states $A^c = MS = MPS = 1$. Given the steady state inflation rate Π and the steady state nominal interest rate Rn, the steady state values of the other variables can be computed as

$$(152) \Rightarrow \qquad \Lambda = \frac{\beta}{1+g} \tag{170}$$

$$(154) \Rightarrow \qquad R = \frac{1}{\Lambda} \tag{171}$$

$$(161) \Rightarrow \qquad \frac{J^c}{JJ^c} = \left(\frac{1-\xi\Pi^{\zeta-1}}{1-\xi}\right)^{\frac{1}{1-\zeta}} \tag{172}$$

$$(159),(160) \Rightarrow \qquad MC = \left(1-\frac{1}{\zeta}\right)\frac{J^c}{JJ^c}\frac{1-\xi\beta\Pi^{\zeta}}{1-\xi\beta\Pi^{\zeta-1}} \tag{173}$$

$$(163) \Rightarrow \qquad \Delta = \frac{(1-\xi)\left(\frac{J^c}{JJ^c}\right)^{-\zeta}}{1-\xi\Pi^{\zeta}} \tag{174}$$

$$(157), \text{using } (150),(151),(155),(158),(162),(164) \Rightarrow \qquad H = \left(\frac{\alpha\Delta MC}{\kappa(1-gy)}\right)^{\frac{1}{1+\phi}} \tag{175}$$

$$(156) \Rightarrow \qquad Y^{W,c} = (A^c H)^{\alpha} \tag{176}$$

$$(162) \Rightarrow \qquad Y^c = \frac{Y^{W,c}}{\Delta} \tag{177}$$

$$G^c = gy * Y^c \tag{178}$$

$$(164) \Rightarrow \qquad C^c = Y^c - G^c \tag{179}$$

$$(159) \Rightarrow \qquad J^c = \frac{Y^c MCMS}{(1-\frac{1}{\zeta})(1-\xi\beta\Pi^{\zeta})} \tag{180}$$

$$(160) \Rightarrow \qquad JJ^c = \frac{Y^c}{(1-\xi\beta\Pi^{\zeta-1})} \tag{181}$$

$$(148) \Rightarrow \qquad U^c = \log(C^c) - \kappa\frac{H^{1+\phi}}{1+\phi} \tag{182}$$

$$(150) \Rightarrow \qquad U^c_{C^c} = \frac{1}{C^c} \tag{183}$$

$$(151) \Rightarrow \qquad U_H = -\kappa H^{\phi} \tag{184}$$

$$(158) \Rightarrow \qquad \frac{P^W}{P} = MC \tag{185}$$

$$(157) \Rightarrow \qquad W^c = \alpha\frac{P^W}{P}\frac{Y^{W,c}}{H} \tag{186}$$

$$(149) \Rightarrow \qquad V^c = \frac{U^c}{1-\beta} \tag{187}$$

Finally we can define

$$
\begin{aligned}
CEquiv_t &= \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(1.01C_{t+s},H_{t+s})\right] - \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(C_{t+s},H_{t+s})\right] \\
&= \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s\left\{\log(1.01C^c_{t+s}) - \kappa\frac{H_{t+s}^{1+\phi}}{1+\phi} - \log(C^c_{t+s}) - \kappa\frac{H_{t+s}^{1+\phi}}{1+\phi}\right\}\right] \\
&= \log(1.01)\sum_{t=s}^{\infty}\beta^s = \frac{\log(1.01)}{1-\beta} \qquad (188)
\end{aligned}
$$

The stationary version should be the same.

B Summary of Behavioural Model Equilibrium

$$
\begin{aligned}
\frac{C^c_t}{1-\beta} &= \frac{Z_t}{[\kappa C^c_t]^{\frac{1}{\phi}}} + ZZ_t && (189) \\
Z_t &= (W^c_t)^{1+\frac{1}{\phi}} + \left(\frac{(1+g)^{1+\frac{1}{\phi}}}{\beta^{\frac{1}{\phi}}}\right)\left(\frac{\mathbb{E}_t\left(Z+\bar{m}_h\hat{Z}_{t+1}\right)}{\mathbb{E}_t\left(\frac{R_n+m_{hr}\hat{R}_{n,t}}{\Pi+\bar{m}_h m_{h\pi}\hat{\Pi}_{t+1}}\right)^{1+\frac{1}{\phi}}}\right) && (190) \\
ZZ_t &= \left(\Gamma^c - T^c + m_{hy}(\hat{\Gamma}^c_t - \hat{T}^c_t)\right) + (1+g)\left(\frac{\mathbb{E}_t\left(ZZ+\bar{m}_h\hat{ZZ}_{t+1}\right)}{\mathbb{E}_t\left(\frac{R_n+m_{hr}\hat{R}_{n,t}}{\Pi+\bar{m}_h m_{h\pi}\hat{\Pi}_{t+1}}\right)}\right) && (191) \\
W^c_t &= \kappa H^\sigma_t C^c_t && (192) \\
W^c_t &= \alpha\frac{P^W_t}{P_t}\frac{Y^{W,c}_t}{H_t} && (193) \\
MC_t &= \frac{P^W_t}{P_t} && (194) \\
Y^{W,c}_t &= A^c_t H^\alpha_t && (195) \\
Y^c_t &= \frac{Y^{W,c}_t}{\Delta_t} && (196) \\
Y^c_t &= C^c_t + G^c_t && (197) \\
G^c_t &= T^c_t && (198) \\
\Gamma^c_t &= Y^c_t - \alpha\frac{P^W_t}{P_t}Y^{W,c}_t && (199) \\
\Delta_t &= \xi\Pi^\zeta_t\Delta_{t-1} + (1-\xi)\left(\frac{J^c_t}{JJ^c_t}\right)^{-\zeta} && (200) \\
J^c_t &= \frac{1}{1-\frac{1}{\zeta}}(Y^c_t)(MC + m_{fmc}\hat{MC}_t)
\end{aligned}
$$

$$
\begin{aligned}
& + \quad \xi \mathbb{E}_t \frac{(\Pi + \bar{m} m_{f\pi} \hat{\Pi}_{t+1})^{\zeta}}{\mathbb{E}_t \left(\frac{R_n + m_{fr} \hat{R}_{n,t}}{\Pi + \bar{m}_f m_{f\pi} \hat{\Pi}_{t+1}} \right)} (J^c + \bar{m}_f \hat{J}^c_{t+1}) && (201)\\
JJ^c_t \quad & = \quad Y^c_t + \xi \mathbb{E}_t \frac{(\Pi + \bar{m} m_{f\pi} \hat{\Pi}_{t+1})^{\zeta-1}}{\mathbb{E}_t \left(\frac{R_n + m_{fr} \hat{R}_{n,t}}{\Pi + \bar{m}_f m_{f\pi} \hat{\Pi}_{t+1}} \right)} (JJ^c + \bar{m}_f \hat{J}J^c_{t+1}) && (202)\\
1 \quad & = \quad \xi \Pi_t^{\zeta-1} + (1-\xi) \left(\frac{J^c_t}{JJ^c_t} \right)^{1-\zeta} && (203)\\
log \left(\frac{Rn_t}{Rn} \right) \quad & = \quad \rho_r log \left(\frac{Rn_{t-1}}{Rn} \right) \\
& + \quad (1-\rho_r) \left[\alpha_\pi log \left(\frac{\Pi_t}{\Pi} \right) + \alpha_y log \left(\frac{Y_t}{Y} \right) + \alpha_{dy} log \left(\frac{Y_t}{Y_{t-1}} \right) \right] \\
& + \quad log(M_t) && (204)\\
log \left(\frac{M_t}{M} \right) \quad & = \quad \rho_m log \left(\frac{M_{t-1}}{M} \right) + \epsilon^m_t && (205)\\
log \left(\frac{G^c_t}{G^c} \right) \quad & = \quad \rho_g log \left(\frac{G^c_{t-1}}{G^c} \right) + \epsilon^g_t && (206)\\
log \left(\frac{MS_t}{MS} \right) \quad & = \quad \rho_{ms} log \left(\frac{MS_{t-1}}{MS} \right) + \epsilon^{ms}_t && (207)\\
log \left(\frac{A^c_t}{A^c} \right) \quad & = \quad \rho_{ma} log \left(\frac{A^c_{t-1}}{A^c} \right) + \epsilon^a_t && (208)\\
& && (209)
\end{aligned}
$$

B.1 Steady State

The exogenous variables have steady states $A^c = MS = M = 1$. Given the steady state inflation rate Π and the steady state nominal interest rate Rn, the steady state values of the other variables can be computed as

$$\Lambda = \frac{\beta}{1+g} \tag{210}$$

$$R = \frac{1}{\Lambda} \tag{211}$$

$$\frac{J^c}{JJ^c} = \left(\frac{1 - \xi \Pi^{\zeta-1}}{1-\xi} \right)^{\frac{1}{1-\zeta}} \tag{212}$$

$$MC = \left(1 - \frac{1}{\zeta} \right) \frac{J^c}{JJ^c} \frac{1 - \xi \beta \Pi^{\zeta}}{1 - \xi \beta \Pi^{\zeta-1}} \tag{213}$$

$$\Delta = \frac{(1-\xi) \left(\frac{J^c}{JJ^c} \right)^{-\zeta}}{1 - \xi \Pi^{\zeta}} \tag{214}$$

$$\frac{P^W}{P} = MC \tag{215}$$

$$H = \left(\frac{\alpha \Delta MC}{\kappa (1 - gy)} \right)^{\frac{1}{1+\phi}} \tag{216}$$

$$Y^{W,c} = (A^c H)^\alpha \tag{217}$$

$$Y^c = \frac{Y^{W,c}}{\Delta} \tag{218}$$

$$\Gamma^c = Y^c - \alpha \frac{P^W}{P} Y^{W,c} \tag{219}$$

$$W^c = \alpha \frac{P^W}{P} \frac{Y^{W,c}}{H} \tag{220}$$

$$G^c = gy * Y^c \tag{221}$$

$$T^c = G^c \tag{222}$$

$$C^c = Y^c - G^c \tag{223}$$

$$J^c = \frac{Y^c MCMS}{(1-\frac{1}{\zeta})(1-\xi\beta\Pi^\zeta)} \tag{224}$$

$$JJ^c = \frac{Y^c}{(1-\xi\beta\Pi^{\zeta-1})} \tag{225}$$

C Estimation Details

C.1 The Measurement Equations

The four observables are: output growth (`dyobs`) defined in various ways, hours worked per capita (`labobs`), monetary policy rate (`robs`), inflation rate (`pinfobs`), real wage growth (`dwobs`). The corresponding measurement equations are:

$$\texttt{dyobs} = \log\left((1+g)\frac{Y_t^c}{Y_{t-1}^c}\right) \tag{226}$$

$$\texttt{labobs} = \frac{H_t - H}{H} \tag{227}$$

$$\texttt{robs} = R_{n,t} - 1 \tag{228}$$

$$\texttt{pinfobs} = \Pi_t - 1 \tag{229}$$

The steady state values of the observables are `dyobs` $= \log(1+g)$, `labobs` $= H$, `robs` $= R_n - 1$, and `pinfobs` $= \Pi - 1$. The estimated parameters $\bar{\Pi}$, $\bar{Rn}$ and $\bar{g}$ are related to the steady state variables of my model by

$$\Pi = \frac{\bar{\Pi}}{100} + 1$$
$$R_n = \frac{\bar{Rn}}{100} + 1$$
$$g = \frac{\bar{g}}{100}$$

From my non-zero-inflation-growth steady state this implies that we should impose the restrictions

$$R_n = \frac{\Pi}{\beta(1+g)^{-1}} = \frac{\bar{Rn}}{100} + 1 \tag{230}$$

on β rather than calibrating it at the usual $\beta = 0.99$. This implies that β is calibrated as

$$\beta = \frac{\frac{\bar{\Pi}}{100} + 1}{\left(\frac{\bar{Rn}}{100} + 1\right)\left(1 + \frac{\bar{g}}{100}\right)^{-1}} \tag{231}$$

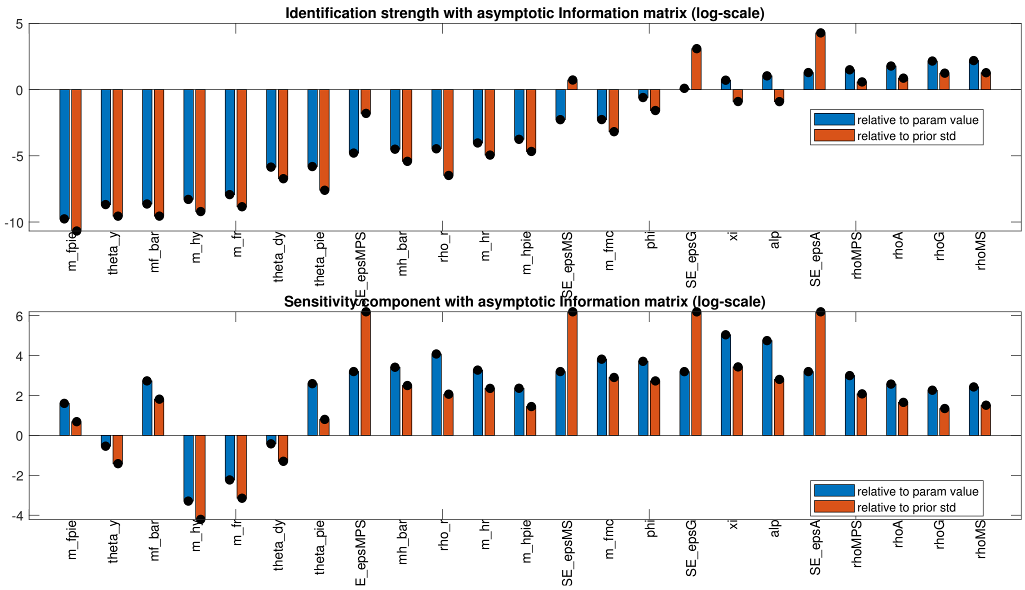

Figure 7: Identification Strength in the tool Model

C.2 Identification

Following Iskrev and Ratto (2010), we provide the identification (locally) analysis of the my tool model here. In the upper panel of the figure the bars depict the identification strength of the parameters based in the Fisher information matrix normalized by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (red bars). Intuitively, the bars represent the normalized curvature of the log likelihood function at the prior mean in the direction of the parameter. If the strength is 0 (for both bars) the parameter is not identified as the likelihood function is flat in this direction. The larger the absolute value if the bars, the stronger the identification. Hence, it is clear that all parameters are identified in the model.

C.3 MCMC Convergence

The convergence property is represented in figure (8). The appended (Interval) shows the Brooks and Gelman's convergence diagnostics for the 80% interval. The blue line shows the 80% interval/quantile range based on the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. The appended (m2) and (m3) show an estimate of the same statistics for the second and third central moments, i.e. the squared and cubed absolute deviations from the pooled and the within-sample mean, respectively. All statistics are based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters. Convergence is indicated by the two lines stabilizing and being close to each other.

The figures from (9) to (11) indicate the prior-posterior plots. The grey line shows the

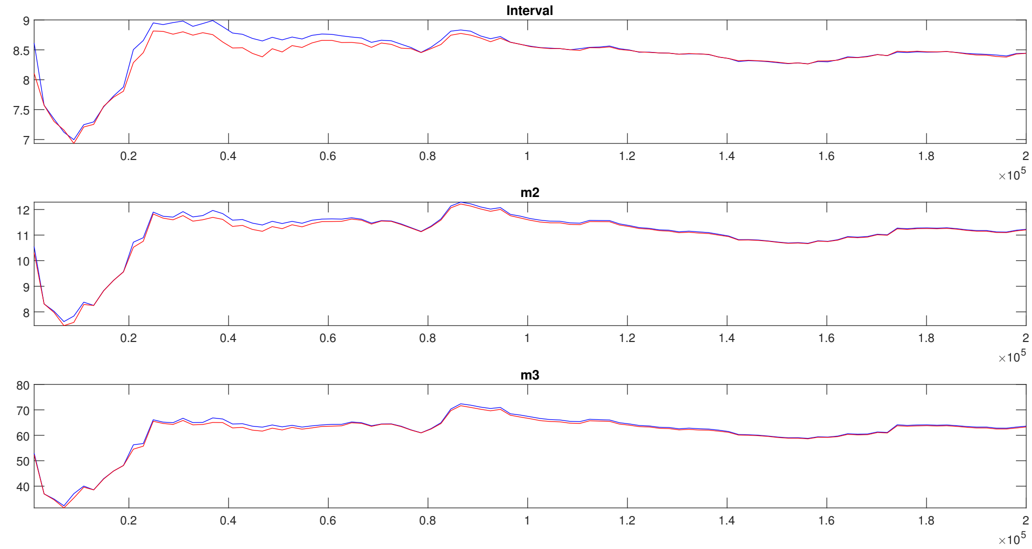

Figure 8: Multivariate convergence diagnostic

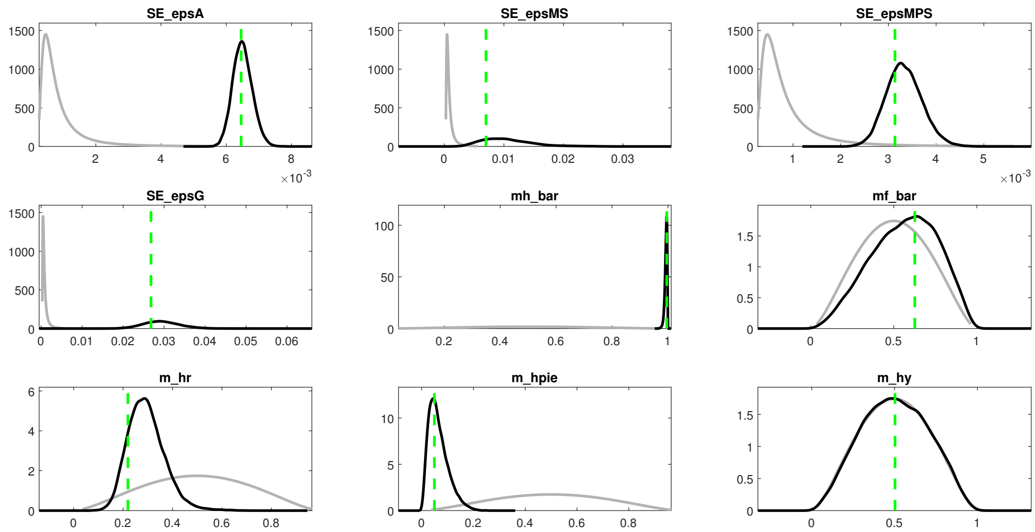

Figure 9: Priors and Posteriors for 200000 MCMC draws

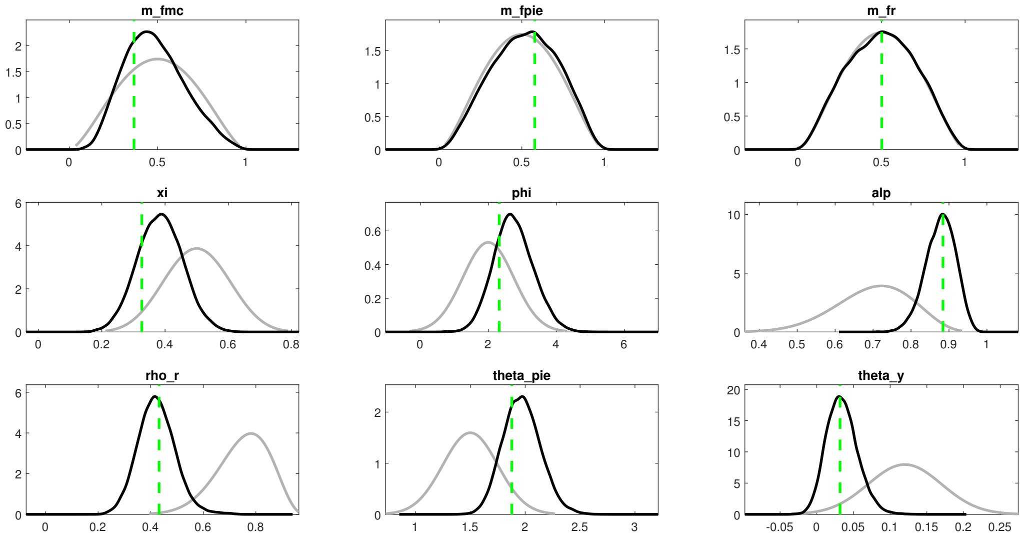

Figure 10: Priors and Posteriors for 200000 MCMC draws

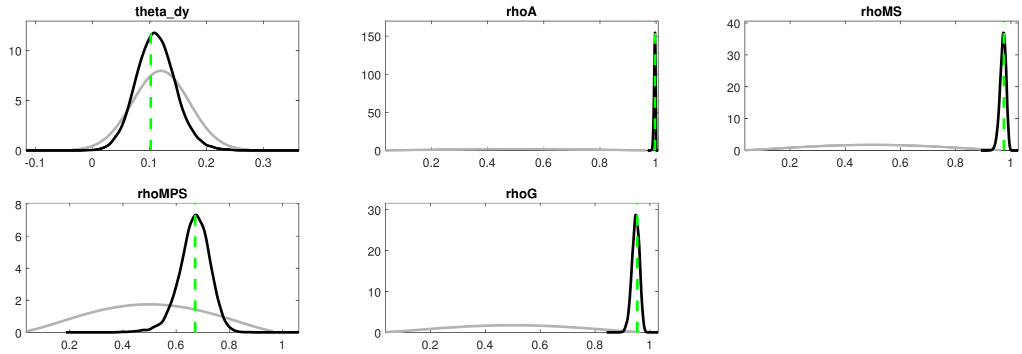

Figure 11: Priors and Posteriors for 200000 MCMC draws

prior density, while the black line shows the density of the posterior distribution. The green horizontal line indicates the posterior mode. If the posterior looks like the prior, either your prior was a very accurate reflection of the information in the data or the parameter under consideration is only weakly identified and the data does not provide much information to update the prior.