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Abstract

This paper examines the conduct of optimal monetary and macro-prudential mandates in a behavioural bounded rationality framework with a financial friction in form of the collateral constraint. A mandate framework is designed where each macroeconomic regulator is assigned a distinct objective function, is instrument-independent and the welfare-optimal parameters of these delegated mandate are computed. Cooperative and non-cooperative mandates games between the monetary and macro-prudential regulation are studied and comparisons drawn between results for rational expectations (RE) and BR models. The main results are: first, in a Bayesian likelihood race the BR model easily outperforms the RE model. Second, the BR model has much larger region of determinacy compared with RE; i.e., the Taylor principle is easier to satisfy under BR. Third, forward guidance is much less powerful under the BR model; i.e., the "forward-guidance puzzle" is resolved. Finally, we confirm a common result in this literature that monetary and macro-prudential cooperation is welfare reducing compared with non-cooperation.

JEL Classification: E52, E58, E61.

Keywords: New Keynesian Behavioural Model, Mandates, Determinacy, Cooperative and Non-cooperative Game

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Contents

1 Introduction

The behavioural model developed in this paper is based on the psychological foundations of bounded rationality brought into the macroeconomic behavioural literature by (Gabaix, 2020) incorporated with a financial friction in form of the collateral constraint (Kiyotaki and Moore, 1997). In particular, agents' representations of the economy are sparse, i.e., when they optimize, agents care only about a few variables that they observe with some myopia. This paper poses a number of questions: what is the welfare-optimal mandate under financial frictions, i.e. should monetary policy focus on traditional inflation and output gap stabilization and/or stabilize financial variables, such as credit spread (and entrepreneur leverage)? What is the role for macro-prudential regulation? Should it be co-ordinated with monetary policy?

We draw on the literature on the financial accelerator studied under rational expectation frameworks and its impact on optimal monetary policies is exhaustive. A short review of this literature follows. However, there are criticisms of policy prescriptions arising from such fully rational-based models. Therefore, revisiting these questions in a behavioural framework is the main exercise of this paper.

1.1 Background Literature

Prior to the financial crisis of 2007–2008, the literature on financial frictions in macroeconomics largely focused on asymmetric information problems and limited contract enforceability (Stiglitz and Weiss (1981), Diamond and Dybvig (1983), Bernanke and Gertler (1989)). In addition, financial frictions emerging from limited commitment took a similar form. For instance, collateral constraints arise in Kiyotaki and Moore (1997) but due to a commitment problem rather than asymmetric information; borrowers cannot commit to repay debt and so must hold collateral as a guarantee. This has an important effect on macroeconomic outcomes as durable goods take on the dual role of being both factors of production and sources of collateral. This dual role creates an accelerator mechanism when the value of capital falls and firm net worth also falls tightening the credit constraint. The reverse is true as the credit constraint slackens during an upturn. Kehoe and Levine (1993) and Cooley et al. (2004) also look at limited contract enforceability and reach similar conclusions.

The collateral constraints approach proposed in Kiyotaki and Moore (1997) has been used to relate fluctuations in real estate prices with economic outcomes in Iacoviello (2005) by assuming that entrepreneurs must post real estate as collateral for loans, and by treating real estate as a factor of production. Here the accelerator mechanism of Kiyotaki and Moore (1997) works via the housing market whereby a fall in house prices would both depress household demand and reduce investment.

Since the recent financial crisis, the number of papers studying the importance of financial frictions on macroeconomic outcomes and policy implications has grown considerably, commonly building on the mechanisms proposed in the Kiyotaki and Moore (1997) (KM) collateral constraints model, or the Bernanke et al. (1999) (BGG) costly state verification model. In KM the propagation and amplification comes from the fluctuations in asset prices, whilst in BGG it originates from from fluctuation of agents net worth.

The KM approach has been extended to study the effects of financial constraints on the banking sector in Gertler and Kiyotaki (2010) (GK) where the limited commitment problem of KM introduces an agency problem between depositors and banks; when the value of bank capital declines, the borrowing constraint tightens and limits the amount of deposits the bank can raise and subsequently, the level of investment. Another extension proposed in Gertler and Karadi (2011) uses this approach to analyse the role of unconventional monetary policy. It is assumed the central bank can perform financial intermediation at a cost, but when the borrowing constraint tightens sufficiently, this cost is less than the inefficiency introduced by the agency problem. The two approaches have both been applied to the housing market. Impatient households post housing as collateral to secure mortgage loans in Iacoviello and Neri (2010) where the mechanism of Iacoviello (2005) is focused on the demand-side of the economy, and shown to have important effects on the business cycle. The collateral constraints arise in Forlati and Lambertini (2011) due to the Bernanke et al. (1999) costly state verification mechanism which is applied to household credit by assuming households observe a private housing-value shock that can lead to default when households are insolvent. The authors emphasise increased housing investment risk in highly leveraged economies.

This paper draws upon both this financial frictions literature and that on NK models with BR. A model with both KM and BGG features provides a tractable framework for the generalization of the Gabaix BR model to incorporate a financial friction faced by an entrepreneur. The reasons for adopting this particular approach to BR are discussed in the review of BR models. In particular, this paper is first related to a strand of the literature on boundedly rational expectation framework (Kreps (1998), Sims (2005), Woodford (2013) and Gabaix (2020)). In general, bounded rationality means agents are inattentive to exogenous variables of interest and future economic conditions. There have been important empirical studies pointing out that agents' expectations are boundedly rational (Coibion and Gorodnichenko (2015)). The majority of the existing literature on behavioural New Keynesian models employs Euler learning, in which agents' decisions are based on first order conditions to maximisation problems. In contrast to the rational expectations solution, in which model consistent expectations enter the first order conditions, Euler learning uses simple bounded rational predictors alongside knowledge of the form of the rational expectations solution. However, a more recent literature on BR models also considers the anticipated utility approach where agents follow an optimal decision rule conditional on their beliefs over aggregate states and prices that are exogenous to their decision variables.[1] This takes into account all information available to the agent, and involves forecasts of variables external to them. The departure of BR (Gabaix (2020) and Woodford (2018)) from aforementioned forms of BR is the use of cognitive discounting, in which agents are unable to fully understand the world, especially the events that are far into the future. In order to capture this behavior Gabaix (2020) assumes that agents, as they simulate the future, the impacts of noisy shocks vanish in the far enough future. As a result, the model converges to the steady state of the economy or the default model.

The main results are: first, in a Bayesian likelihood race the BR model easily outperforms the RE model. Second, the BR model has much larger region of determinacy compared with RE; i.e., the Taylor principle is easier to satisfy under BR. Third, forward guidance is much less powerful under the BR model; i.e., the “forward-guidance puzzle" is resolved. Finally, we confirm a common result in this literature that monetary and macro-prudential cooperation is welfare reducing compared with non-cooperation.

1.2 Roadmap

The rest of the paper is structured as follows. Section 2 describes the general modelling of the policy game in this paper. Section 3 sets out the model which is estimated in Section 4. Section 5 studies two properties of the model: first its ability to address the forward guidance puzzle and second, the saddle-path stability of the monetary rule. Section 6 describes the policy framework in the form of a delegation game. The main results of the paper are those for the policy mandates and are set out in Section 7. Section 8 conclude the paper.

2 Methodology: A Linear Quadratic Framework

Deak et al. (n.d.) use a non-linear modelling framework and only makes a second-order perturbation approximation when it comes to the solution of the stochastic steady state. By contrast, this paper formulates a linear-quadratic (LQ) approximation to the non-linear dynamic optimization problem from the outset. This LQ approach (also used by Debortoli et al. (2019)) is widely used for a number of reasons. First, for LQ problems the characterization of time-consistent (discretionary) and commitment equilibria for a single policy maker, and even more so for many interacting policymakers, are well understood. Although this paper does not consider discretionary equilibrium, it does formulate compare cooperative and non-cooperative regimes between the monetary and

[1] The anticipated utility approach with infinite time horizons is also referred the *infinite-time horizon* approach (Deak et al., 2015). Bounded rationality of this form can be generalized to finite time horizons - see Woodford (2018).

macro-prudential regulator. Second, the certainty equivalence property results in optimal rules that are robust in the sense that they are independent of the variance-covariance matrix of additive disturbances. Third, policy can be decomposed into deterministic and stochastic components. This is a very convenient property since it enables the stochastic stabilization component to be pursued using simple Taylor-type feedback rules rather than the exceedingly complex optimal counterpart.

Judd (1998), pages 507-509, draws attention to a general Hamiltonian framework for approximating a nonlinear problem by an LQ one due to Magill (1977*a*), who appears to be the first to have applied it in the economics literature.[2] This paper was the precursor to a literature led by Michael Woodford that considers an LQ approximation to the Ramsey problem in the context of DSGE models (see Woodford (2003), and Benigno and Woodford (2012)) for one-country models and Benigno and Benigno (2006) for a two-country generalization.

Whereas Judd's emphasis is on the perturbation approach which focuses on computing derivatives of the nonlinear optimization problem Levine et al. (2008) is about replacing various nonlinear problems in a one-country and two-country context with ones that are LQ. It should be noted that the Judd first-order perturbation approximation of the decision rule and the Hamiltonian approach generate the *same* LQ approximation. Levine et al. (2008) develops the Magill framework in presenting a discrete-time version of his results generalized to rational expectations models with forward-looking variables. Their results include second-order necessary conditions for non-concave intertemporal problem; second, extends the Hamiltonian approach to a comparison between cooperative and non-cooperative equilibria. The latter is particularly relevant for the dynamic game studied in this Paper.

3 The Model

This section derives a linearized form of the model with the added feature of a KM form of a financial friction. The household and price-setting firms' decisions are unchanged but are included for completeness. The financial friction is faced by an entrepreneur and this sub-section is the additional feature of the model. We choice a simple form of financial friction that combines features from KM and BGG. As in KM the source of the financial friction is a collateral constraint; as in BGG there is a separate agent, the entrepreneur, who faces this constraint. Although other forms of a financial friction as reviewed in Section 1.1 are possible and possibly have more empirical support (see for example, Deak et al. (2019)), this set-up proves to be a particularly tractable framework to introduce Gabaix-type myopia and inattention.

[2] See also, Magill (1977*b*).

3.1 Household

Household's single-period utility is

$$U(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\sigma}}{1+\sigma}$$

In a stochastic environment, the value function of the representative household at time t is given by

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}, N_{t+s}) \right] \ ; \ \beta \in (0,1) \tag{1}$$

where C_t is consumption, N_t is the labour supply and all variables are expressed in real terms relative to the price of retail output.

For the household's problem at time t is to choose paths for consumption $\{C_t\}$, labour supply, $\{N_t\}$, capital stock $\{K_t\}$ and investment $\{I_t\}$ to maximize V_t given by (1) given its budget constraint in period t with all variables in real terms:

$$B_t = R_t B_{t-1} + r_t^K K_{t-1} + W_t H_t + \Gamma_t - I_t - T_t - C_t \tag{2}$$

where B_t is the stock of one-period bonds at the end of period t, r_t^k is the rental rate for capital, W_t is the wage rate and R_t is the interest rate set in period $t-1$ paid in period t on bonds held at the beginning of period t. Note K_t is *end-of-period* capital stock. And Γ_t is the profit transfered from firms.

So that the real household's income is:

$$Y_t^h = r_t^k K_{t-1} + W_t H_t + \Gamma_t - I_t - T_t$$

Then the budget constraint (2) becomes:

$$B_t = R_t B_{t-1} + Y_t^h - C_t \tag{3}$$

Capital stock accumulates according to

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t \, ; && (4) \\ X_t &\equiv \frac{I_t}{I_{t-1}} \, ; \ S', \, S'' \geq 0 \, ; \ S(1) = S'(1) = 0 && (5) \end{aligned}$$

(5), $S(X_t)$ are investment adjustment costs, I_t units of output converts to $(1 - S(X_t))I_t$ of new capital sold at a real price Q_t. The FOC solution with respect to labour supply N_t is given by

$$W_t = \frac{N_t^{\sigma}}{C_t^{-\gamma}} \tag{6}$$

and capital producer solution is

$$Q_t(1 - S(X_t) - X_t S'(X_t)) + \mathbb{E}_t\left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2\right] = 1 \tag{7}$$

The behaviour household maximizes the same value function as in (1) but does not pay full attention to all the variables in the budget constraint (8), as correctly processing information entails a cost. The behavioural agent perceives reality with some myopia, which is associated with deviations from the steady state real interest rate, $\hat{R}_t = R_t - \bar{R}$, and real income, $\hat{Y}_t^h = Y_t^h - \bar{Y}^h$

The linearized behavioural agent's budget constraint is

$$B_t = (\bar{R} + m_r \hat{R}_t) B_{t-1} + (\bar{Y}^h + m_y \hat{Y}_t^h) - C_t \tag{8}$$

Where m_r and m_y are the inattention parameters in [0,1] with $m_r = m_y = 1$, we have the full rational household's BC. If the future state vector of the whole economy populated by rational agents evolves as

$$S_{t+1} = f(S_t, \epsilon_{t+1}) \tag{9}$$

Then the future state vector of the whole economy populated by behavioural agents is

$$S_{t+1} = \bar{m} f(S_t, \epsilon_{t+1}) \tag{10}$$

Where $\bar{m}$ in [0,1] is the general myopia of the agent regarding the economy's state. Rational case when $\bar{m} = 1$.

The linearized Euler equation of the behavioural household is given by:

$$\tilde{C}_t = \frac{\bar{m}}{R - (R-1)m_Y} E_t[\tilde{C}_{t+1}] - \frac{m_r}{\gamma R(R - (R-1)m_Y)} E_t[\hat{R}_{t+1}] \tag{11}$$

where $R = \frac{1}{\beta}$. Hence, if the vector of myopia parameters $(\bar{m}, m_r, m_Y) = (1,1,1)$ (fully rational expectation case), the Euler equation above becomes the standard Euler equation in the fully rational case.

3.2 Entrepreneur

A risk-averse entrepreneur purchases capital from capital producers at a price Q_t and rents it to wholesale producers. The entrepreneur consumes in every period and can raise her net worth by lowering her consumption. Her single-period utility is

$$U_{E,t} = \frac{C_{E,t}^{1-\gamma_E}}{1-\gamma_E}$$

In a stochastic environment, the value function of the representative entrepreneur at time t is given by

$$V_{E,t} = \mathbb{E}_t\left[\sum_{s=0}^{\infty} \beta_E^s U(C_{E,t+s})\right] \;;\; \beta_E \in (0,1) \tag{12}$$

Where $\gamma_E \geq 0$. The discounted rate $\beta_E \leq \beta$ for households captures a probability of exit for the entrepreneur.

The entrepreneur is subject to two constraints. The first is a budget constraint in real terms

$$L_t = R_{L,t}L_{t-1} + Q_tK_t - R_t^K Q_{t-1}K_{t-1} + C_{E,t} - T_{E,t} \tag{13}$$

The second and crucial constraint of the entrepreneur is the collateral constraint in the real term:

$$E_t[R_{L,t+1}]L_t \leq \phi_t E_t[Q_{t+1}]K_t \tag{14}$$

Where ϕ_t is the tightness level of the collateral constraint. There is uncertainty about the entrepreneur's collateral constraint on its expectation about the loans' and capital's prices. The FOCs of the above maximization problem is represented in the appendices A-C.

The behaviour entrepreneur maximizes the same value function as in (12) but does not pay full attention to all the variables in the budget constraint (13) and collateral constraint (14), as correctly processing information entails a cost. The behavioural agent perceives reality with some myopia, which is associated with deviations from the steady state of the exogenous variable to herself. In particular, these variables are current real interest rate on loan: $\hat{R}_{L,t} = R_{L,t} - R_L$, future real interest rate on loan: $\hat{R}_{L,t+1} = R_{L,t+1} - R_L$, real return on capital: $\hat{R}^K_t = R^K_t - R^K$, current capital price: $\hat{Q}_t = Q_t - Q$, future capital price : $\hat{Q}_{t+1} = Q_{t+1} - Q$, and on the real transfer $\hat{T}_{E,t} = T_{E,t} - T_{E,t}$. Notice that, entrepreneur pays different levels of attention on the exogenous variable today and that tomorrow which enter in her constraints.

The behavioural agent's budget constraint is:

$$L_t = [\bar{R_L} + m_{RL1}\hat{R}_{L,t}]L_{t-1} + [Q + m_{Q1}\hat{Q}_t]K_t - [R^K + m_{RK}\hat{R}^K_t][Q + m_{Q1}\hat{Q}_{t-1}]K_{t-1} + C_{E,t} - (T_E + m_T\hat{T}_{E,t}) \tag{15}$$

Similarly, the behavioural agent's budget constraint is:

$$E_t[R_L + m_{RL2}\hat{R}_{L,t+1}]L_t \leq \phi_t E_t[Q + m_{Q2}\hat{Q}_{t+1}]K_t \tag{16}$$

The value function of the optimization problem is

$$V_E(L_t, K_t) = max_{c_E} U_E(C_E) + \beta_E V_E(L_{t+1}, K_{t+1})$$

We now have the linearized behavioural entrepreneur's consumption function:

$$\begin{aligned}
\tilde{C}_{E,t} &= \frac{\beta_E \phi(m_{RL2} - m_{RL1}) + (\phi/R_L)m_{RL2}}{\gamma_E(R^K - \phi)[1 - (1 - \beta_E)m_T]} E_t[\tilde{R}_{L,t+1}] \\
&- \frac{\beta_E R^K m_{RK}}{\gamma_E(R^K - \phi)[1 - (1 - \beta_E)m_T]} E_t[\tilde{R}^K_{t+1}] \\
&- \frac{(\beta_E R^K - 1)m_{Q1}}{\gamma_E(R^K - \phi)[1 - (1 - \beta_E)m_T]} \tilde{Q}_t \\
&+ \frac{(\beta_E R^K - 1)m_{Q2}}{\gamma_E(R^K - \phi)[1 - (1 - \beta_E)m_T]} E_t\tilde{Q}_{t+1} \\
&+ \frac{(\beta_E R^K - 1)}{\gamma_E(R^K - \phi)[1 - (1 - \beta_E)m_T]} \tilde{\phi}_t \\
&+ \frac{\beta_E \bar{m}}{1 - (1 - \beta_E)m_T} E_t[\tilde{C}_{E,t+1}]
\end{aligned} \tag{17}$$

Again, when the vector of myopia parameters $[m_{RL1}, m_{RL2}, m_{Q1}, m_{Q2}, m_{RK}, m_T, \bar{m}] = [1]$ we have the consumption function of the fully rational expectation entrepreneur. The economy's demand curve is the combination of equation (11) and equation (17) and the market clearing condition.

$$\tilde{Y}_t = \frac{C}{Y}\tilde{C}_t + \frac{C_E}{Y}\tilde{C}_{E,t} + \frac{I}{Y}\tilde{I}_t + \frac{G}{Y}\tilde{G}_t \tag{18}$$

3.3 The behavioural non-zero steady state inflation Phillips Curve

Details on the derivation of the log-linearized non-zero steady state inflation Phillips Curve are presented in the appendices. Therefore, we just show the final form the behavioural Phillips curve which is a linearized form.

$$\tilde{J}_t = (1-\beta\xi\Pi^{\zeta})\left(\tilde{Y}_t + m_{fmc}\tilde{MC}_t\right) + \beta\xi\Pi^{\zeta}\mathbb{E}_t\left((1+\zeta)\bar{m}_f m_{f\pi}\tilde{\Pi}_{t+1} - m_{fr}\tilde{R}_{n,t} + \bar{m}_f\tilde{J}_{t+1}\right) \tag{19}$$

$$\tilde{JJ}_t = (1-\beta\xi\Pi^{\zeta-1})\tilde{Y}_t + \beta\xi\Pi^{\zeta-1}\mathbb{E}_t\left(\zeta\bar{m}_f m_{f\pi}\tilde{\Pi}_{t+1} - m_{fr}\tilde{R}_{n,t} + \bar{m}_f\tilde{JJ}_{t+1}\right) \tag{20}$$

$$\tilde{\Pi}_t = \frac{1-\xi\Pi^{\zeta-1}}{\xi\Pi^{\zeta-1}}(\tilde{J}_t - \tilde{JJ}_t) \tag{21}$$

Where ξ is the Calvo's price parameter, ζ is the elasticity of substitution, $\tilde{MC}_t$ is the marginal cost, $\tilde{Y}_t$ is the aggregate output, all tilded variables are in log-linearized form.

Notice that the expectation terms here are fully rational, the vector of myopia parameters included in the set of equations above represents the behavioural element of the boundedly rational price-setting firms. When the steady state of inflation is zero (or the steady state gross inflation $\Pi = 1$), we can directly subtract equation (20) from equation (19) and substitute into equation (21) to eliminate $\tilde{Y}_t$ and $\tilde{Rn}_t$ to get a standard Phillips curve at the zero steady state level of inflation as follows:

$$\tilde{\Pi}_t = \frac{(1-\xi)(1-\beta\xi)m_{fmc}}{\xi}\tilde{MC}_t + \beta\left[(1-\xi)\bar{m}_f m_{f\pi} + \xi\bar{m}_f\right]\mathbb{E}_t\tilde{\Pi}_{t+1} \tag{22}$$

Again, we can retreat the Phillips curve of the fully rational price-setting firm if the vector of myopia parameters, $[\bar{m}, m_{fmc}, m_{f\pi}, m_{fr}]$, is equal to the vector of 1. Although our behavioural Phillips curve (with zero steady state inflation) is isomophic to that of Gabaix (2020), our behavioural Phillips curve also has the same property as Gabaix's which is less forward-looking compared to the fully rational case. In other word, when firms are more attentive to the macroeconomic outcomes, say, vector m is closer to one, then firms are more forward-looking because the slope on future inflation is higher. This property is well preserved in case of non-zero steady state inflation.

3.4 Monetary and macro-prudential policy rules

We now close the model by describing the monetary and macro-prudential rules. First we assume that the central bank employs a Taylor-type rule for the nominal interest rate as follows:

$$\tilde{R}_{n,t} = \rho_r \tilde{R}_{n,t-1} + (1-\rho_r)(\theta_\pi \tilde{\Pi}_t + \theta_{dy}(\tilde{Y}_t - \tilde{Y}_{t-1})) + \epsilon_t^r \tag{23}$$

where $\rho^{rn} \in (0,1)$ is the inertia parameter, $\theta^\pi \leq 0$ is feedback parameter on inflation, and $\theta^{dy} \leq 0$ is feedback parameter on output growth.

In general, the Loan-To-Value (ϕ_t, LTV) ratio is an exogenous parameter which is not endogenously affected by model's behaviors. However, in this framework we use the LTV ratio as a way to control the degree of financial frictions. When the LTV ratio is high, the collateral constraint is less tight. Lowering the LTV tightens the constraint and therefore restricts the loans that enterpreneurs can obtain. As a result, we propose a Taylor-type rule for the LTV ratio which reacts adversely to the growth rate of loans.[3]

$$\tilde{\phi}_t = \theta_l(\tilde{L}_t - \tilde{L}_{t-1}) + \epsilon_t^\phi \tag{24}$$

Where $\theta_l \leq 0$ is a feedback parameter on the credit growth. Negative value of θ_l means that when there is an excess in credit growth the macro-prudential authority tends to lower LTV, or tightening the collateral constraint.

This completes the description of the behavioural NK model with a financial friction for a given setting of monetary and macro-prudential instruments. The full equilibrium is summarized in Appendix C.

4 Estimation

In this section, we estimate a subset of deep parameter and the inattention parameter vector of the behavioural model. The fully rational expectation model is also estimated to compare the performance of the two model in term of matching the data.

[3] In the spirit of the Basel III regulation which aims at avoiding episodes of excessive credit growth.

In a linear setup the model can be characterized by a standard transition equation:

$$s_t = A(p)s_{t-1} + B(p)u_t \tag{25}$$

and a measurement equation:

$$y_t = C(p) + D(p)s_t \tag{26}$$

where $u_t = (\epsilon_t^a,\ \ \epsilon_t^g,\ \ \epsilon_t^{ms},\ \ \epsilon_t^m,\ \ \epsilon_t^\phi,\ \ \epsilon_t^{IS})$ is a vector of structural shocks, s_t is a vector of stationary variables, and $y_t = (\Delta ln(GDP),\ \Delta ln(Investment),\ ln(Labour),$ $nominal{-}rate, inflation, rates{-}spread)$ is a vector of observables. $A(p), B(p), C(p), D(p)$ are matrices of reduced form parameters.

4.1 Estimation results

Bayesian techniques are employed with the standard Kalman Filter employed to calculate the likelihood for a given sample of data of the observable vector. The same data is employed as in Deak et al. (n.d.). Results are presented in the table 1. Appendices E.1 and E.2 provide details of identification and MCMC convergence tests.

Parameters	Prior			Post. RE		Post. BR	
	pdf	Mean	Std	Mode	s.d.	Mode	s.d.
Technology shock (ϵ_a)	**IG**	0.0010	0.02	0.0074	0.0005	0.0070	0.0004
Mark-up shock (ϵ_{ms})	**IG**	0.0010	0.02	0.0292	0.0027	0.0253	0.0058
Monetary policy shock (ϵ_m)	**IG**	0.0010	0.02	0.0020	0.0002	0.0012	0.0001
Gov spending shock (ϵ_g)	**IG**	0.0010	0.02	0.0479	0.0039	0.0165	0.0010
Financial shock (ϵ_{ϕ_E})	**IG**	0.0010	0.02	0.1444	0.0142	0.0183	0.0025
Investment shock (ϵ_{KQ})	**IG**	0.0010	0.02	0.0022	0.0004	0.0016	0.0002
Technology Persistence (ρ_a)	**IG**	0.50	0.20	0.9847	0.0042	0.9789	0.0029
Mark-up Persistence (ρ_{ms})	**IG**	0.50	0.20	0.9313	0.0176	0.9379	0.0242
Gov. spending shock Persistence (ρ_{ms})	**IG**	0.50	0.20	0.9827	0.0044	0.9847	0.0021
Investment shock Persistence (ρ_{ms})	**IG**	0.50	0.20	0.6372	0.0784	0.9700	0.0089
Calvo's parameter(ξ)	**B**	0.50	0.20	0.8785	0.0050	0.7780	0.0457
Investment's adj cost parameter(ξ)	**N**	2	0.75	4.7108	0.3769	4.8296	0.4833
Lag interest rate, rule(ρ_r)	**B**	0.50	0.10	0.6847	0.0262	0.9110	0.0137
Inflation's, rule (θ_π)	**N**	2.0	0.25	1.4814	0.0668	1.7245	0.1505
Output growth's, rule(θ_{dy})	**N**	0.120	0.050	0.3010	0.0297	0.1625	0.0487
Loan growth's para, rule (θ_l)	**N**	-0.15	0.25	-0.5799	0.2592	-1.9220	0.1275
Inattention parameters	**Prior**			**Post. RE**		**Post. Behaviour**	
$\bar{m}$ myopia parameter general state	**B**	0.8	0.15			0.2206	0.0887
mr Household's myopia parameter on nor. interest	**B**	0.80	0.15			0.4575	0.2056
my Household's myopia parameter on income	**B**	0.80	0.15			0.9456	0.1120
m_{RL1} Entrepreneur's myopia parameter on loan's rate	**B**	0.80	0.15			0.0235	0.0118
m_{RL2} Entrepreneur's myopia parameter on exp loan's rate	**B**	0.80	0.15			0.8109	0.2289
m_{RK} Entrepreneur's myopia parameter on capital's rate	**B**	0.80	0.15			0.2303	0.0076
m_{Q1} Entrepreneur's myopia parameter on capital price	**B**	0.80	0.15			0.9125	0.0526
m_{Q2} Entrepreneur's myopia parameter on exp cap price	**B**	0.80	0.15			0.9950	0.0100
m_T Entrepreneur's myopia parameter on transfer	**B**	0.80	0.15			0.8325	0.2765
m^f_π Firm's myopia parameter on inflation	**B**	0.80	0.15			0.9346	0.1332
m^f_{mc} Firm's myopia parameter on marginal cost	**B**	0.80	0.15			0.7064	0.3586
m^f_r Firm's myopia parameter on real interest rate	**B**	0.80	0.15			0.9460	0.1114
m^f_y Firm's myopia parameter on agg. output	**B**	0.80	0.15			0.9338	0.1346

Table 1: Estimation results - Parameter

Our estimated results show that the boundedly rational expectation reduces the scale of structural price-stickiness friction, ξ, and the magnitude of estimated shocks, which, most importantly, improves the marginal likelihood relative to the RE model. Smets and Wouters (2003) and Smets and Wouters (2007) have shown that rational models with a rich set of frictions and a general stochastic structure can explain the data relatively well. However, these models require an implausibly high level of price and wage stickiness and exogenous shocks to explain the observed persistence in the data. In particular, from table (1), estimated shocks and persistence parameters are significantly higher than that under BR model. In addition, the estimated feedback parameter on the macroprudential rule is much smaller than that under RE model, which illustrates that based on the data the macroprudential authority under BR environment has higher level of countercyclicality to the growth rate of the credit compared to case under RE. Finally, agents's myopia to the future states, $\bar{m}$, of the economy is significantly high.

	RE (1)	BR (2)
LM (1981Q1-2017Q1)	2484.68	3027.53

Table 2: Log marginal data density (LM), based on Laplace approximation

From the Log marginal data density results in table 2, the BR model outperforms the RE counterpart. In other word, the RE model is firmly rejected by the data in favour of the BR model.

4.2 Second Moment Comparisons with Data

Sims (2001) address some concerns with Bayesian model comparison and the importance of seeking alternative criteria. In particular, he argues when using a single evidence in favor of a particular characteristic of the model while ignoring other factors can lead to disparate inference. In other words, the Bayesian model comparison is criticized on the basis of the argument that the models considered are too sparse. In such cases, posterior odds may lead to extreme outcomes. In addition, it is well known characteristic of the Bayesian approach when the estimated results are sensitive to the prior distributions. To further evaluate the absolute performance of one particular model against data, in this section we compare the models' implied characteristics (covariances and autocorrelations) with those of the actual data.

	Standard Deviation					
Model	Output growth	Inflation	Interest rate	Investment	Worked hours	Spread Rate
Data	0.0062	0.0026	0.0079	0.0189	0.0410	0.0039
Model RE	0.1343	0.0072	0.0119	0.0744	0.1552	0.6027
Model BR	0.0106	0.0133	0.0180	0.0482	0.1427	0.0214
	Cross-correlation with Output growth					
Data	1.0	0.0814	0.2626	0.6959	0.1602	-0.1557
Model RE	1.0	0.2218	0.6699	0.8001	0.6170	0.9959
Model BR	1.0	0.3196	-0.0659	0.9218	0.2101	0.7368
	Autocorrelations (Order=1)					
Data	0.4082	0.6028	0.9674	0.6573	0.9823	0.9348
Model RE	-0.5455	0.7356	0.1641	-0.0571	0.2227	-0.5665
Model BR	0.7557	0.9657	0.9944	0.8917	0.9917	0.7570

Table 3: Selected Second Moments of the Model Variants

Table 3 presents some selected second moments implied by the above estimations and compares with those in the actual data. In particular, we compute these model-implied statistics by solving the models at the posterior means obtained from estimation. The results of the model's second moments are compared with the second moments in the actual data to evaluate the models' empirical performance. Specifically, the BR model performs relatively better than the RE model in matching actual data's statistics, such as standard deviation, and cross-correlation with output, but both models perform relatively poorly in some dimensions. However, the RE model does a better job regarding the inflation's moments.

To further illustrate how the estimated models capture the data statistics and persistence in particular, we now plot the autocorrelations up to order 10 of the actual data and those of the endogenous variables generated by the model variants in Figure (1). Overall, the behavioural model does a better job at matching the persistence of actual data, especially regarding output, nominal interest rate, investment and worked hours series. However, the RE model does a much better job at matching with the inflation data.

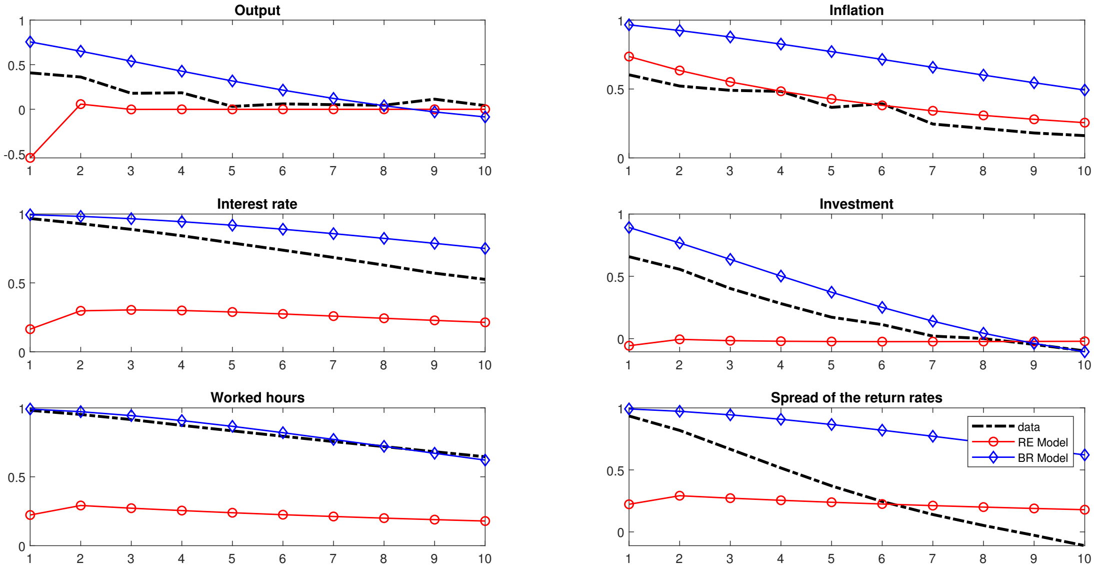


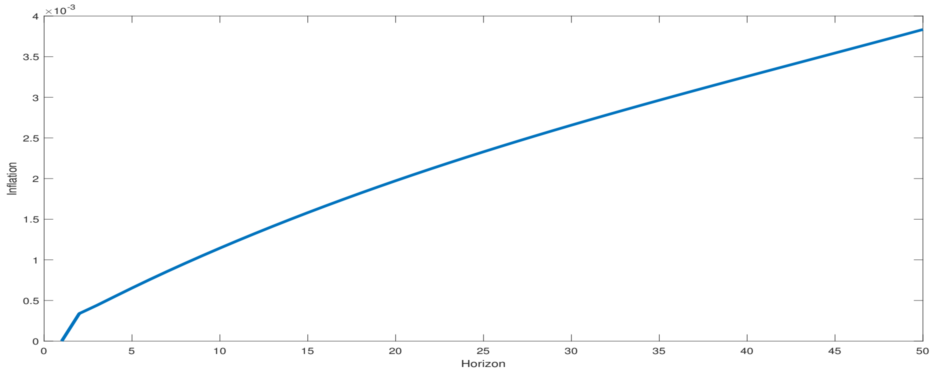
Figure 1: Autocorrelations of Observables in the Actual Data and in the Estimated Models. Sample period from 1981Q1 to 2017Q4 (Estimated sample)

5 Properties of the Estimated Model: Forward Guidance and Determinacy

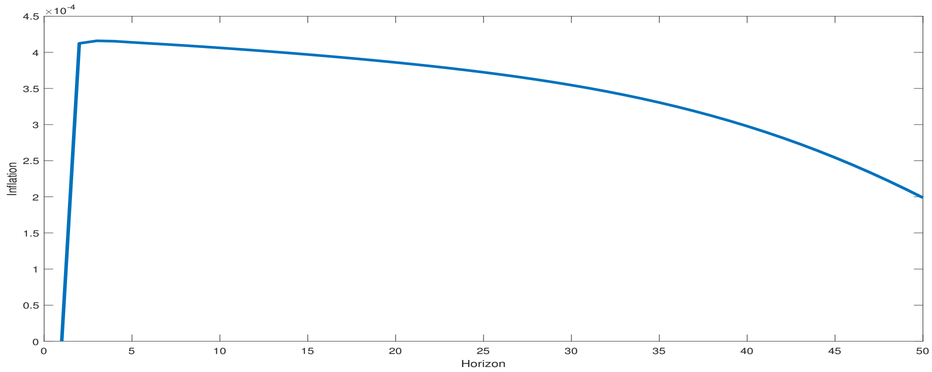
This section uses the estimated model to examines two issues: first the ability of the BR model to resolve the “forward-guidance puzzle". Second, the stability and determinacy of the Taylor-type monetary rule in the RE and BR two models: in particular how the available saddle-path stable parameter space for the policy rule change when we move from RE to BR?

5.1 Forward guidance

Suppose the central bank announces at time 0 that in T periods it will perform a one-period, 1 percent real interest rate cut. What is the impact on today’s inflation? Figure (2) illustrates the effect. In the upper panel, the whole economy is rational. We see that the further away the policy, the bigger the impact today – this is quite surprising, hence the term ”forward guidance puzzle”. In the lower we have the result from the BR model. We see that indeed, announcements about very distant policy changes have small effects



(a) Rational Expectation



(b) Rational Inattention

Figure 2: This Figure shows the response of current inflation to forward guidance about a one-period interest rate cut in T quarters, compared to an immediate rate change of the same magnitude. Upper panel: traditional RE model. Lower panel: model with BR. Parameters are the same in both models, except for the myopia parameters which are equal to 1 in the RE model.

with behavioral agents – but they have the biggest effect with rational agents.[4]

5.2 In/determinacy of the simple rule.

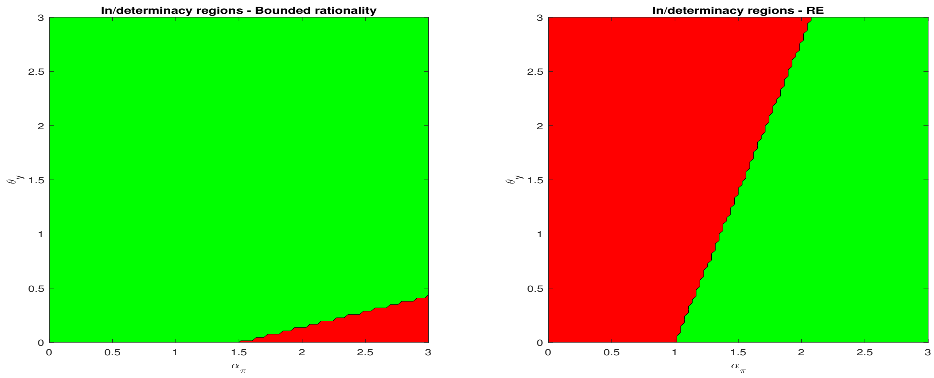
assuming that the central bank sets the nominal interest it in a Taylor rule fashion:

$$\tilde{R}_{n,t} = \rho_r \tilde{R}_{n,t-1} + \theta_\pi \tilde{\Pi}_t + \theta_y \tilde{Y}_t + \epsilon_t^m \tag{27}$$

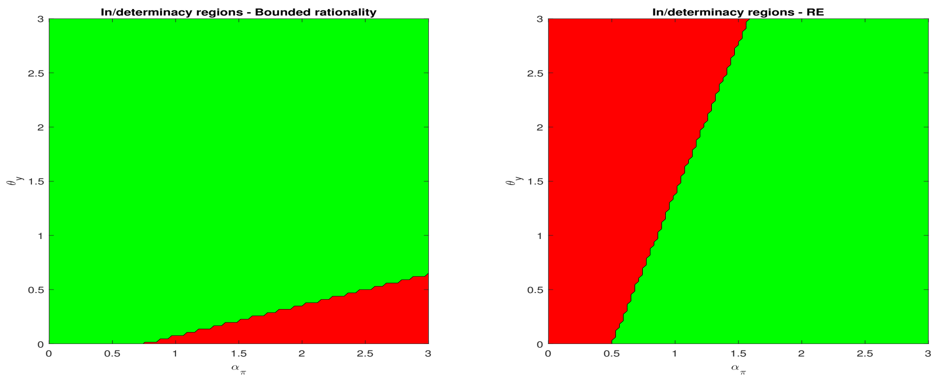
Where ϵ_t^m is an exogenous shock. From panel (a) of figure (3), the standard Taylor principal is prevailed, i.e. $\theta_\pi \geq 1$, under the estimated RE model. In panel (b) is the case of BR

[4]This result can be derive directly by solving the rational Phillips curve forward

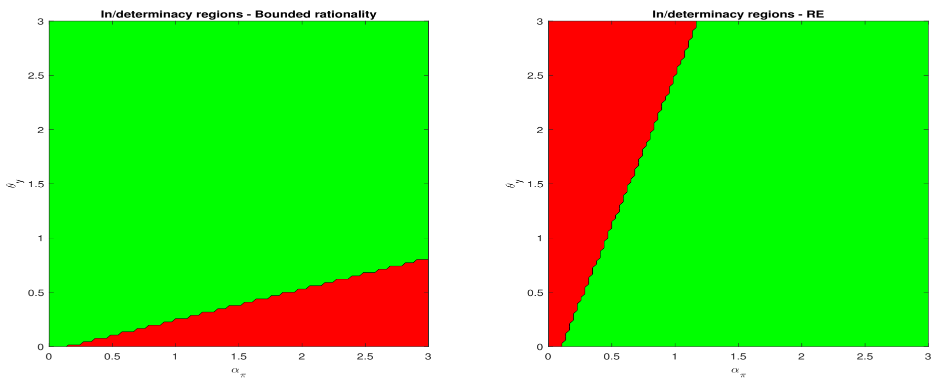
model, the estimated BR model has unique solution even when the nominal interest rate is constant, i.e. $\theta_\pi = \theta_y = 0$. Under the estimated BR model, the estimated value of the general myopic parameter, $\bar{m}$, is very close to zero, which results in a much flatter Phillips Curve. Hence, the condition that the eigenvalues of the system are less than 1 can be achieved with a smaller set of θ_π.



(a) $\rho_r = 0$



(b) $\rho_r = 0.5$



(b) $\rho_r = 0.8$

Figure 3: This Figure shows the determinacy region (Green area) and indeterminacy region (Red area) of the simple in the following form: $\tilde{R}_{n,t} = \rho_r \tilde{R}_{n,t-1} + \theta_\pi \tilde{\Pi}_t + \theta_y \tilde{Y}_t$.

6 Optimal Monetary and Macro-Prudential Mandates

This section sets out the more details of the framework for examining Monetary and Macro-Prudential Mandates. We first of all describe the implementation of the LQ methodology set out in Section 2. We then formalize policy in terms of a dynamic delegation game.

6.1 LQ approximation of policy problem

Following Section 2 and Debortoli et al. (2019) we first consider the central bank's decision problem from an optimal perspective. Consider the Ramsey policy with the society's utilitarian objective function derived from the sum of the household's and entrepreneur's utility functions. Let X_t be a state vector describing at time t all the macroeconomic variables in the model needed to define these utilities in the quadratic approximation.

$$\Omega_t(X_t) = \sum_{t=0}^{\infty} \beta^t \left(U(X_t) + U_E(X_t)\right) \tag{28}$$

The purely quadratic form of the society utility function is then derived as:

$$\Omega_t(X_t) \simeq \text{Constant} - \mathbb{E}_t \sum_{t=0}^{\infty} \left[\beta^t X_t' W^{society} X_t\right] \tag{29}$$

where the constant term is invariant in this linear framework because it only depends on the steady state of the economy. Hence, the Ramsey planner at time t chooses $X_t = X_t^*$ to implement the following optimal policy problem:

$$X_t^* \left(W^{society}\right) = -\text{argmin}_{(X_t)} \mathbb{E}_t \sum_{t=0}^{\infty} \left[\beta^t X_t' W^{society} X_t\right] \tag{30}$$

resulting in a welfare for society:

$$\Omega_t^{society} = -\mathbb{E}_t \sum_{t=0}^{\infty} \left[\beta^t \left[\left(X_t^*(W^{society})\right)' W^{society} \left(X_t^*(W^{society})\right)\right]\right] \tag{31}$$

However, consistent with our mandates approach and the practice of most central banks, transparent mandate should only involve a few variables. This is then designed in the form of the following two-stage delegation game.

6.2 The Delegation Game

Stage 2: The central bank objectives: Assume that the central bank is delegated a simple loss function of the following from:

$$\mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta^t X_t' W^{CB} X_t\right] \tag{32}$$

where W^{CB} is a positive definite matrix that transforms $X_t' W^{CB} X_t$ into a transparent simple loss function. Then given this simple mandate, optimal policy of the central bank becomes:

$$X_t^*\left(W^{CB}\right) = -\text{argmin}_{(X_t)}\mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta^t X_t' W^{CB} X_t\right] \tag{33}$$

In general, if $W^{society} \neq W^{CB}$ we have that $X_t^*\left(W^{society}\right) \neq X_t^*\left(W^{CB}\right)$. The central banks' simple mandate then specifies only the objective variables, but it at this stage we are ambiguous about the weights attached to these variables.

Stage 1: Designing the optimal mandates: Given the form of the weighting matrix, W^{CB}, the benevolent policymaker designs an optimal form of the mandate to solve the optimization problem:

$$\Omega_t^{society} = -\text{argmin}_{(W^{CB})}\sum_{t=0}^{\infty}\mathbb{E}_t\left[\left(X_t^*(W^{CB})\right)' W^{society}\left(X_t^*(W^{CB})\right)\right] \tag{34}$$

with the welfare outcome:

$$\Omega_t^{society} = -\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}\left[\left(X_t^*(W^{CB})\right)' W^{society}\left(X_t^*(W^{CB})\right)\right]\right] \tag{35}$$

The game is, then, solved by backward induction in the standard way.

This then defines *Ramsey mandate* for the central bank which differs from the Ramsey planner's problem in the previous sub-section. In the subsequent analysis the Ramsey mandate is used as a benchmark against which to assess the welfare costs of the optimized simple Taylor-type rules for the policymakers in the subsequent analysis.

7 Numerical Results

7.1 A Ramsey mandate policy.

We first write down the form of our general mandate:

$$\Omega_t^{CB} \equiv -\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^\tau\left(\tilde{\Pi}_{t+\tau}^2+\lambda_y\tilde{Y}_{t+\tau}^2+\lambda_{rn}\tilde{R}_{n,t+\tau}^2+\lambda_l\tilde{L}_{t+\tau}^2+\lambda_\phi(\phi_{t+\tau}^E)^2\right)\right]$$
$$\equiv -\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^\tau(X_t^{\prime}W^{CB}X_t)\right] \tag{36}$$

with weights to be determined. (36) then delegates a mandates which targets variances of inflation, output (deviation from the flexible economy), the nominal interest rates, loan credit market and the loan-to-capital ratio.

	Rational Expectations							**Bounded Rationality**						
	Optimal weights					Welfare (CEV)	P_{zlb}	Optimal weights					Welfare (CEV)	P_{zlb}
Regimes	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	λ_ϕ^*	–	–	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	λ_ϕ^*	–	–
$\Pi = 1.0$	1	0.57	0.0	0.13	0.0	-1.634 (-0.45 %)	0.485	1	0.15	0.0	0.0	0.0035	-1.186 (0 %)	0.48
$\Pi = 1.003$	1	0.3	0.0	0.13	0.0	-1.976 (-0.79 %)	0.475	1	0.086	0.0	0.0	0.0032	-1.385 (-0.20 %)	0.47
$\Pi = 1.005$	1	0.2	0.0	0.12	0.0	-2.129 (-0.95 %)	0.468	1	0.066	0.0	0.0	0.0512	-1.530 (-0.35 %)	0.46
$\Pi = 1.01$	1	0.08	0.0	0.095	0.0	-2.30 (-1.11 %)	0.45	1	0.03	0.0	0.0	0.03	-1.95 (-0.76 %)	0.44
$\Pi = 1.015$	1	0.03	0.0	0.06	0.0	-2.32 (-1.13 %)	0.43	1	0.01	0.0	0.0	0.02	-2.46 (-1.27 %)	0.43

Table 4: Optimal weights on endogenous variables of the delegated Ramsey mandate.

Table 4 shows the results of the optimal mandates under different formations of the expectations, namely rational expectations (RE) and bounded rationality (BR) with different levels of long-run inflation target, Π. Overall, in this Ramsey Mandate including the financial variables into the macroeconomic authorities' objectives leads to either zero weights or very small weights. With the RE model, inflation, output are welfare optimal. In contrast with the BR model, inflation, output and loan-to capital are welfare optimal. However, the relative weight attached on output is decreasing with the levels of long-run inflation target. In both cases however the higher the level of steady state inflation the more volatile the model is, the central bank put a significantly higher relative weight on inflation compare with output variations. The probability of nominal interest rate zero lower bound episodes falls slightly. However we use the zero net inflation BR model without zero lower such bound considerations as the welfare benchmark against which to assess the optimized simple rule mandates which now follow.

7.2 The optimized simple rule mandates.

As in the mandate in the NK models of Deak et al. (n.d.), the central bank uses the Taylor rule to minimize the loss function which contains the variability of inflation and output. However, the objective function of the economy now also includes financial variables with the presence of the financial frictions in the model. Following the financial crisis 2008, it is generally accepted that the stabilizing the financial aspects of the market is crucial to achieve desirable macroeconomic policies' outcomes. Therefore, the question of how these macroeconomic authorities coordinate in achieving distinct macroeconomic objectives has featured in recent literature.

In order to contribute to the discussion we consider three cases. First, the central bank acts as a single macroeconomic authority whose objective function contains of the inflation, output, and nominal interest variability with a Taylor-type rule of nominal interest rate as an instrument. Second, we compute the optimal monetary and macro-prudential policies when there is coordination between the central bank and the macro-prudential regulator. Finally, a non-coordinated game in a non-cooperation equilibrium between these players is considered.

7.2.1 Optimal simple rule mandate without the Macro-prudential Authority.

We first consider a benchmark case in which there is only a monetary authority that acts in the traditional way having been delegated a mandate in the following form:

$$\Omega_t^{CB} \quad = \mathbb{E}_t\Big[\textstyle\sum_{s=0}^{\infty}\beta^s\Bigg(\tilde{\Pi}_{t+s}^2 + \lambda_y \tilde{Y}_{t+s}^2 + \lambda_{rn}\tilde{R}_{n,t+s}^2\Bigg)\Big] \tag{37}$$

and using the interest rate as an instrument:

$$\tilde{R}_{n,t} = \rho_r \tilde{R}_{n,t-1} + (1-\rho_r)(\theta_\pi \tilde{\Pi}_t + \theta_{dy}(\tilde{Y}_t - \tilde{Y}_{t-1})) \tag{38}$$

This is then a mandate in the form of a objective (37) and an interest rate rule (38). Table

	Steady state inflation level is equal to 0 %							
	Rational Expectations				**Bounded Rationality**			
	Optimal weights			Welfare (CEV)	Optimal weights			Welfare (CEV)
Optimal weights	λ^*_π	λ^*_y	λ^*_{rn}	–	λ^*_π	λ^*_y	λ^*_{rn}	–
	1.0	1.32	0.014	-8.023 (-6.84 %)	1.0	0.20	0.38	-10.22 (-9.03 %)
Optimal feedback params	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}
	0.03	1.54	1.16	0.50	0.63	1.53	1.14	0.43
	Steady state inflation level is equal to 1.2 %							
	Fully rational expectation				**Behavioural setup**			
	Optimal weights			Welfare (CEV)	Optimal weights			Welfare (CEV)
Optimal weights	λ^*_π	λ^*_y	λ^*_{rn}	–	λ^*_π	λ^*_y	λ^*_{rn}	–
	1.0	1.06	0.0054	-8.77 (-7.58 %)	1.0	4.49	4.1	-11.676 (-10.49 %)
Optimal feedback params	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}
	0.15	1.54	1.16	0.49	0.29	1.65	5.11	0.37
	Steady state inflation level is equal to 2 %							
	Fully rational expectation				**Behavioural setup**			
	Optimal weights			Welfare (CEV)	Optimal weights			Welfare (CEV)
Optimal weights	λ^*_π	λ^*_y	λ^*_{rn}	–	λ^*_π	λ^*_y	λ^*_{rn}	–
	1.0	3.93	0.022	-9.36 (-8.17 %)	1.0	0.13	0.0079	-12.61 (-11.42 %)
Optimal feedback params	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}
	0.23	1.52	1.133	0.49	0.75	4.68	5.24	0.34
	Steady state inflation level is equal to 4 %							
	Fully rational expectation				**Behavioural setup**			
	Optimal weights			Welfare (CEV)	Optimal weights			Welfare (CEV)
Optimal weights	λ^*_π	λ^*_y	λ^*_{rn}	–	λ^*_π	λ^*_y	λ^*_{rn}	–
	1.0	1.24	0.0	-11.24 (-10.05 %)	1.0	5.6	1.3	-16.00 (-14.81 %)
Optimal feedback params	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}
	0.4	1.52	1.13	0.49	0.69	4.33	5.45	0.20
	Steady state inflation level is equal to 6 %							
	Fully rational expectation				**Behavioural setup**			
	Optimal weights			Welfare (CEV)	Optimal weights			Welfare (CEV)
Optimal weights	λ^*_π	λ^*_y	λ^*_{rn}	–	λ^*_π	λ^*_y	λ^*_{rn}	–
	Indeterminacy				1.0	3.13	1.9	-20.63 (-19.44 %)
Optimal feedback params	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}	ρ^*_r	θ^*_π	θ^*_{dy}	P_{zlb}
	Indeterminacy				0.75	4.66	5.26	0.064

Table 5: Optimal monetary policy under optimized simple rule at different levels of steady state inflation.

5 presents the results.[5]

In these computations a shift in the steady state inflation rate is used to lower the probability of the nominal interest rate hitting the zero lower bound constraint. However we have seen that in the RE model the parameter space for which the rule is saddle-path stable is much smaller than that for the BR model. As a result a higher steady state

[5] The interest rate was computed with a term in the level of output, but when optimized this turned out to be almost zero.

inflation eventually induces indeterminacy for the RE model but successfully lowers the probability zero lower bound episodes for the BR case. Then the optimized rule shows similar properties to those the NK models with financial frictions of Deak et al. (n.d.).

Overall, the most striking feature of the results is a substantial welfare loss of optimized simple rule mandates compared to the case of Ramsey mandates. To take the case of the BR model a low zero-lower-bound probability of 0.2 and 0.064 is achieved at annual trend inflation rates of 4% and 6% respectively at a welfare CEV cost of 16% and around 21%. This is largely the welfare cost of these high steady state inflation rates, but even with zero net inflation the CEV cost is around 10%. So why are such large costs occurring? The reason must lie in the constraint imposed by the simple rule which is absent in the Ramsey mandate where the form of the rule implicit in the solution is unconstrained. We have a model with a financial constraint that alters substantially the choice of capital by the entrepreneur in the model. In this world of second best the Ramsey setting of the nominal interest rate (and the macro-prudential instrument considered next) is inevitably a highly complex form of rule that departs substantially from the simple rule imposed here. This suggests better rules could be explored that still are simple to implement. We return to this theme in the conclusions of the paper.

7.2.2 A cooperative game.

We now consider the cooperative objective function for the macro-prudential (MP) and monetary authorities. In this context, we assume that they are delegated an identical objective function as follows:

$$\Omega_t^{CB,MP} = -\mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\left(\tilde{\Pi}_{t+s}^2+\lambda_y\tilde{Y}_{t+s}^2+\lambda_{rn}\tilde{R}_{n,t+s}^2+\lambda_l\tilde{L}_{t+s}^2+\lambda_\phi\tilde{\phi}_{t+s}^2\right) \tag{39}$$

where λ_y, λ_{rn} are weights attached to monetary target variables, which are relative to inflation's weight. Similaly, λ_l, and λ_ϕ are relative weights attached to financial sector's variables.

The macro-prudential and monetary authorities use their instruments to minimize the delegated welfare loss function. These again take the form of Taylor-type rules which replace the implied complex instrument settings in the Ramsey mandate. The monetary policymaker (the central bank) uses a nominal interest rule

$$\tilde{R}_{n,t} = \rho_r \tilde{R}_{n,t-1} + (1-\rho_r)(\theta_\pi \tilde{\Pi}_t + \theta_{dy}(\tilde{Y}_t - \tilde{Y}_{t-1})) \tag{40}$$

and the macro-prudential authority employs the rule on $\tilde{\phi}_t$, which governs the tightness level of the collateral constraint.

$$\tilde{\phi}_t = \theta_l(\tilde{L}_t - \tilde{L}_{t-1})) \tag{41}$$

However, in a cooperative manner, we have to assume that each macroeconomic authority is fully informed about the optimal policy decision of other macroeconomic authority. In other word, the problem above can be re-written as minimizing the objective function (39) given the combined rules (40) and (41).

Table 6 shows the results of the mandate under optimized simple rules. Overall, in term of welfare comparison, mandates under optimized simple rules again result in significant welfare loss compared to the cases under Ramsey.

There are other striking results from table 6. First, comparing to the case of Ramsey mandate, under both BR and RE, optimized simple rule mandates suggest that the macroeconomic authorities should attach a large weight on the financial variables, namely the loans' and loan-capital ratio's variations. Second, optimized simple monetary rules converge to an extended Taylor-type rules with very high weights on inflation and output growth. In addition, under RE, the smoothing parameter is very close to zero, which leads to a high probability of the nominal interest rate hitting the ZLB across different levels of steady state inflation. To reduce this probability the monetary policymaker again imposes a higher steady state inflation.

7.2.3 A non-cooperative fame in a closed-loop Nash Equilibrium.

In the NK model with both fiscal and monetary policy conducted independently, in Stage 3 we need a Closed-loop Nash Equilibrium (CLNE) in the optimized feedback coefficients. First, we redefine the objective function of the macro-prudential and monetary authorities in stage 3. The macroeconomic authorities objective functions are defined as:

	Steady state inflation level is equal to 0 %											
	Rational Expectations						**Bounded Rationality**					
	Optimal weights					Welfare (CEV)	Optimal weights					Welfare (CEV)
Optimal weights	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–
	1.0	6.34	3.44	1.88	4.92	-8.023 (-6.84 %)	1.0	2.97	0.06	1.15	3.22	-10.21 (-9.02 %)
Optimal feedback params	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}
	0.0	4.75	5.28	-4.464	-	0.4958	0.71	4.80	5.23	-4.48	-	0.4312
	Steady state inflation level is equal to 1.2 %											
	Fully rational expectation						**Behavioural setup**					
	Optimal weights					Welfare (CEV)	Optimal weights					Welfare (CEV)
Optimal weights	λ_π^*	λ_y^*	λ_{Rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–	λ_π^*	λ_y^*	λ_{Rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–
	1.0	4.13	4.47	7.93	1.67	-8.814 (-7.63 %)	1.0	0.96	0.39	4.74	0.19	-11.55 (-10.36 %)
Optimal feedback params	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}
	0.014	5.52	4.45	-4.40	–	0.4937	0.73	4.33	5.54	-4.7	–	0.3789
	Steady state inflation level is equal to 2 %											
	Fully rational expectation						**Behavioural setup**					
	Optimal weights					Welfare (CEV)	Optimal weights					Welfare (CEV)
Optimal weights	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–
	1.0	1.01	0.08	1.58	0.08	-9.390 (-8.20 %)	1.0	4.13	0.43	4.25	0.75	-12.61 (-11.42 %)
Optimal feedback params	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}
	0.14	1.52	1.12	-4.0	–	0.4925	0.74	4.8	5.22	-4.48	-	0.3386
	Steady state inflation level is equal to 4 %											
	Fully rational expectation						**Behavioural setup**					
	Optimal weights					Welfare (CEV)	Optimal weights					Welfare (CEV)
Optimal weights	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–
	1.0	1.12	3.1	8.08	0.62	-11.31 (-10.12 %)	1.0	8.43	0.83	8.57	3.51	-15.95 (-14.76 %)
Optimal feedback params	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}
	0.3	4.55	5.38	-4.365	–	0.1693	0.79	1.54	1.16	-4.0	-	0.220
	Steady state inflation level is equal to 6 %											
	Fully rational expectation						**Behavioural setup**					
	Optimal weights					Welfare (CEV)	Optimal weights					Welfare (CEV)
Optimal weights	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	$\lambda_{\phi_E}^*$	–
	-	-	-	-	-	- (- %)	1.0	5.49	2.02	0.39	0.095	-20.61 (-19.42 %)
Optimal feedback params	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	–	P_{zlb}
	-	-	-	-	–	-	0.77	1.47	1.05	-3.996	-	0.0683

Table 6: Cooperative game between monetary and macro-prudential authorities under optimized simple rule at different levels of steady state inflation.

$$\Omega_t^{CB} \equiv \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\left(\tilde{\Pi}_{t+s}^2 + \lambda_y(\tilde{Y}_{t+s} - \tilde{Y}_{t+s}^F)^2 + \lambda_{rn}\tilde{R}_{n,t+s}^2\right) \tag{42}$$

$$\Omega_t^{MP} \equiv \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\left(\lambda_l\left(\tilde{L}_{t+s}\right)^2+\lambda_\phi\tilde{\phi}_{t+s}^2\right) \tag{43}$$

Here we denote Ω_t^j, where $j = [Macro - prudential(MP), \quad monetary(CB)]$, are the welfare function for each policy maker. Each policy maker has her own policy instrument, $int_{j,t}$. The monetary policy maker uses nominal interest rule

$$\tilde{R}_{n,t} = \rho^r\tilde{R}_{n,t-1} + (1-\rho_r)(\theta_\pi\tilde{\Pi}_t + \theta_{dy}(\tilde{Y}_t - \tilde{Y}_{t-1})) \tag{44}$$

and the macro-prudential authority employs the rule on $\tilde{\phi}_t$, which governs the tightness level of the collateral constraint:

$$\tilde{\phi}_t = \theta_l(\tilde{L}_t - \tilde{L}_{t-1}) \tag{45}$$

The delegation game proceeds as follows:

Stage 2 macroeconomic authorities maximize their own objective functions subjected to the policy rules above. In particular, CB uses the simple rule on nominal interest rate to maximize its objective function in form of (42) taking into account the optimized macro-prudential rule (45), and vice verse.

Stage 1 the social planner designs the optimal mandates ((42) and (42)) delegated to the CB and the macro-prudential authority by maximizing the true social welfare function in (29) given the closed-loop game between macroeconomic authorities and the constraints in **Stage 2**.

Results of the bounded rationality setup.

Table 7 sets out the results of this non-cooperative game. In term of welfare performance, the non-cooperative policy between macroeconomic authorities produces a higher welfare compared to the case of cooperative game. Again, RE outperforms BR in terms of welfare performance. In addition, financial variables are significant in the delegated mandates. Finally, the optimized simple rules under both BR and RE place a high weight on financial variable feedback parameter, especially, under BR this weight is unbounded from below, which means the central bank is extremely aggressive in countering the credit growth's

	Results of the BR setup											
	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	λ_ϕ^*	$-$	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	Welfare (CEV)	P_{zlb}
$\Pi = 1.0$	1.0	5.9	1.6	2.9	3.9		0.0	2.23	1.01	-100	-9.58 (-8.39 %)	0.42
$\Pi = 1.003$	1.0	3.67	4.15	3.66	5.24		0.0	3.14	0.74	-100	-10.49 (-9.30 %)	0.36
$\Pi = 1.005$	1.0	6.15	6.79	4.74	4.38		0.0	3.23	1.02	-100	-11.17 (-9.98 %)	0.30
$\Pi = 1.01$	1.0	9.13	0.98	5.47	9.65		0.0	4.0	0.75	-100	-12.84 (-11.65 %)	0.14
$\Pi = 1.015$	1.0	8.46	1.36	4.06	9.69		0.0	5.0	1.0	-100	-15.54 (-14.35 %)	0.01
	Results of the RE setup											
	λ_π^*	λ_y^*	λ_{rn}^*	λ_l^*	λ_ϕ^*	$-$	ρ_r^*	θ_π^*	θ_{dy}^*	θ_l^*	Welfare (CEV)	P_{zlb}
$\Pi = 1.0$	1.0	1.89	3.14	2.44	5.0		0.29	2.14	0.99	-7.8	-6.03 (-4.84 %)	0.494
$\Pi = 1.003$	1.0	7.49	2.71	2.42	5.16		0.32	4.79	0.86	-9.0	-6.06 (-4.87 %)	0.491
$\Pi = 1.005$	1.0	5.70	2.96	1.04	2.64		0.33	4.85	1.0	-9.67	-6.11 (-4.92 %)	0.489
$\Pi = 1.01$	1.0	1.39	1.94	1.67	5.5		0.30	2.12	0.98	-10.5	-6.35 (-5.16 %)	0.486
$\Pi = 1.015$	1.0	3.72	4.84	0.83	0.6		0.33	4.85	1.0	-9.67	-6.85 (-5.66 %)	0.482

Table 7: Non-cooperative game under optimized simple rule. (CE = 0.01)

volatility. Moreover, the smoothing parameter of the monetary policy simple rule is small.

Comparing Tables 6 and 7, in term of welfare performance between cooperative and non-cooperative games, the striking result here is that the non-cooperative game actually results in a *higher* welfare gain compared to the case of cooperative macroeconomic policy manner. Generally in a world of second-best counter-productive cooperation is a possible outcome. In particular, Svensson (2012) argues that letting each regulator focus on its own objective, leads to a more effective performance in reducing volatilities. In particular, macro-prudential policy acts in a much more aggressive way, favoring the reduction of the volatility of loan growth. Rubio and Carrasco-Gallego (2014) also find that the non-cooperative manner generates higher welfare gain than under the cooperative between macroeconomic regulators is also the results using an ad-hoc objective function approach. The results from tables 6 and 7 show that the optimal weights attached on macroeconomic variables in the objective functions are also higher in the non-coordinated manner, which shows that lack of coordination makes the regulators to be more aggressive in achieving their own objective. Finally, form of expectation matters for the welfare outcome. In particular, under the Ramsey mandate, BR model outperforms the RE one, but the opposite is true for the simple rule mandate.

8 Conclusions

The model developed in this paper extends and estimates the behavioural NK model of Gabaix (2020) to incorporate a financial friction in form of the collateral constraint in Kiyotaki and Moore (1997). We adopt a LQ approach to the policy problems and study a Ramsey mandates, and both cooperative and non-cooperative simple rule mandates.

The main results are: The main results are: first, in a Bayesian likelihood race the BR model easily outperforms the RE model. In a second moments validation exercise the BR model also comes closer to reproducing the second moments of the data. Secondt, the BR model has much larger region of determinacy compared with RE , i.e., the Taylor principle is easier to satisfy under BR. Third, forward guidance is much less powerful under the BR model; i.e., the “forward-guidance puzzle" is resolved. Fourth, we confirm a common result in this literature that monetary and macro-prudential cooperation is welfare reducing compared with non-cooperation.

A striking result is that our optimized simple rules do not come close to mimicking the Ramsey Mandate in terms of welfare. This suggests an important avenue for further research along to directions. First, the form of the nominal interest rate rule chosen by the central bank and the loan-to-capital instrument by the macro-prudential regulator need developing to match better the implied complex rules in the Ramsey Mandate. Second, a further unconventional monetary policy instrument in the form of liquidity injections can be added which enables the collateral constraint to at least partially by-passed. Another area for future research would be to develop the underlying behavioural model of financial frictions drawing upon the large literature surveyed in the Introduction of this Paper.

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Appendices

A Model

A.1 The Behavioural Household's consumption function

Deriving the Taylor expansion of the consumption deviations: The Lagrangian of the problem described is

$$\mathbb{L} = \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t^b (-B_t + R_t B_{t-1} + Y_t^h - C_t) \tag{46}$$

where $R_t = \bar{R} + m_r \hat{R}_t$,$Y_t^h = \bar{Y}^h + m_y \hat{Y}_t^h$, λ_t^b is the Lagrangian multiplier, which is equal to $V_b(B_t)$, the derivation of the value function w.r.t B_t. The value function of the optimization problem is

$$V(B_t) = max_c U(C) + \beta V(B_{t+1})$$

At the optimum, the agent solves the following problem $V(B) = max_{C,B} L$. The Envelope theorem implies that.

$$V_{R_{t+1}} = L_{R_{t+1}} = \beta^t [U_R(C_t) + \beta \lambda_{t+1}^b B_t] \tag{47}$$

Taking the total derivatives this expression w.r.t B_o we get

$$V_{R_t, B_o} = \beta^t \frac{D(B_t)}{D(B_o)} D(B_t)[U_r(C_t) + \beta \lambda_{t+1}^b B_t] \tag{48}$$

Following Gaibax (2013): In this consumption problem, such as $\beta = \frac{1}{R}$, under the default model of $C_t = C_o$, $B_t = B_o$, so $\frac{D(B_t)}{D(B_o)} = 1$, $U_{R_t}(C) = 0$, $V_{B_t}^d = \lambda_t^b = U'(C) = C_t^{-\gamma}$, and finally $C_t = \frac{(R-1)B_t+\bar{Y}}{R}$. Hence, we can derive the one-time change of $\hat{R}_t$.

$$\begin{aligned}
V_{R_t,B_o} &= \beta^t D(B_t)\left[\beta\left(\frac{(R-1)B_{t+1}+R\bar{Y}^h}{R}\right)^{-\gamma} B_t\right] \\
&= \frac{1}{R^{t+1}}\left[-\gamma\frac{R-1}{R}C_{t+1}^{-\gamma-1}\frac{D(B_{t+1})}{D(B_t)}B_t + C_{t+1}^{-\gamma}\right] \\
&= \frac{1}{R^{t+1}}C_o^{-\gamma-1}\left[-\gamma\frac{(R-1)}{R}B_o + C_o\right] \qquad (49)
\end{aligned}$$

Again, under the default model $C_t = C_o$. As time-0 consumption satisfies $U_{C_o} = V_{B_o}$, taking derivatives of both sides w.r.t $\hat{R}_t$ we have $U_{C,C}\partial_{\hat{R}_t}C_o = \partial_{\hat{R}_t}V_{B_o} = V_{R_t,B_o}$. Hence,

$$\partial_{\hat{R}_t}C_o = \frac{\partial_{\hat{R}_t}V_{B_o}}{U_{C,C}} = \frac{1}{R^{t+1}}\left(\frac{(R-1)}{R}B_o - \frac{C_o}{\gamma}\right) \qquad (50)$$

So that we have:

$$b_R(B_t) = \frac{1}{R^{t+1}}\left(\frac{(R-1)}{R}B_o - \frac{C_o}{\gamma}\right) \qquad (51)$$

We now take the derivative of the value function w.r.t Y_t^h. Similarly, applying the Envelop theorem yields

$$V_{Y_t^h} = L_{Y_t^h} = \beta^t[U_{Y^h}(C_t) + \lambda_t^b] \qquad (52)$$

Taking the total derivatives this expression w.r.t B_o we get

$$V_{Y_t^h,B_o} = \beta^t\frac{D(B_t)}{D(B_o)}D(B_t)[U_{Y^h}(C_t) + \lambda_t^b] \qquad (53)$$

Which can be simplified by using the default model's assumptions

$$V_{Y_t^h,B_o} = \frac{1}{R^t}\left[-\gamma\frac{(R-1)}{R}C_o^{-\gamma-1}\right] \qquad (54)$$

Similarly, we have $U_{C,C}\partial_{\hat{Y}_t^h}C_o = \partial_{\hat{Y}_t^h}V_{B_o} = V_{Y_t^h,B_o}$. Hence,

$$\partial_{\hat{Y}_t^h}C_o = \frac{\partial_{\hat{Y}_t^h}V_{B_o}}{U_{C,C}} = \frac{1}{R_{t+1}}(R-1) \qquad (55)$$

The Taylor expansion for $\hat{C}_t$ is:

$$\hat{C}_t = E_t\sum_{\tau\geq t+1}\frac{b_{R|B=0}\hat{R}_{\tau+1}}{R^{\tau-t+1}} + E_t\sum_{\tau\geq t}\frac{b_Y\hat{Y}_\tau^h}{R^{\tau-t+1}} \qquad (56)$$

where $b_R = \frac{(R-1)}{R^2}B_o - \frac{C_o}{R\gamma}$, and $b_Y = \bar{r}$.
Hence, for the behavioural agent expression, (56) becomes

$$\hat{C}_t = E_t^{BR} \sum_{\tau \geq t} \frac{b_{R|B=0}\hat{R}_{\tau+1} + b_Y \hat{Y^h}_\tau}{R^{\tau-t+1}} \tag{57}$$

Following Gabaix (2016) the term structure of attention: $E_t^{BR}[\hat{R}_{t+k+1}] = m_r \bar{m}^k E_t[\hat{R}_{t+k+1}]$ and $E_t^{BR}[\hat{Y}_{t+k}^h] = m_Y \bar{m}^k E_t[\hat{Y}_{t+k}^h]$, replacing these expressions into the equation (57) we get:

$$\hat{C}_t = E_t \sum_{\tau \geq t} \frac{\bar{m}^{\tau-t}}{R^{\tau-t+1}} \left(b_{R|B=0} m_r \hat{R}_{\tau+1} + b_Y m_Y \hat{Y}_\tau^h \right) \tag{58}$$

Dividing equation (58) by $\bar{C}$, we get

$$\frac{\hat{C}_t}{\bar{C}} = E_t \sum_{\tau \geq t} \frac{\bar{m}^{\tau-t}}{R^{\tau-t+1}} \left(\frac{b_{R|B=0}}{\bar{C}} m_r \hat{R}_{\tau+1} + b_Y m_Y \frac{\hat{Y}_\tau^h}{\bar{C}} \right) \tag{59}$$

Using the market clearing condition we must have $Y_t^h = C_t$, thus $\frac{\hat{C}_t}{\bar{C}} = \frac{\hat{Y}_t^h}{\bar{C}} = \tilde{Y}_t$, and $\frac{b_{r|B=0}}{\bar{C}} = \frac{1}{\bar{C}} \left(\frac{-C_o}{\gamma} \right) = -\frac{1}{\gamma}$. Hence, the equation (59) becomes

$$\tilde{C}_t = E_t \sum_{\tau \geq t} \frac{\bar{m}^{\tau-t}}{R^{\tau-t+1}} \left(-\frac{1}{R\gamma} m_r \hat{R}_{\tau+1} + (R-1) m_Y \tilde{C}_\tau \right) \tag{60}$$

By expanding the IS curve above we have consumption function as in the equation (**??**).

$$\tilde{C}_t = -\frac{1}{R^2\gamma} m_r E_t[\hat{R}_{t+1}] + \frac{(R-1)}{R} m_Y \tilde{C}_t + \frac{\bar{m}}{R} E_t[\tilde{C}_{t+1}] \tag{61}$$

By simplifying the equation above we have the behavioural household's consumption as the equation (11).

A.2 The Behavioural Entrepreneur's consumption function

We first work out the FOCs of the fully rational expectation entrepreneur or the maximization problem in the main text. The Lagrangian is :

$$\begin{aligned} \mathbb{L}_E = E_t \sum_{t=0}^{\infty} \beta_E^t U(C_{E,t}) \quad &+ \quad E_t \sum_{t=0}^{\infty} \beta_E^t \lambda_{E,t}^1 (L_t - R_{L,t} L_{t-1} - Q_t K_t + R_t^K Q_{t-1} K_{t-1} - C_{E,t} + T_{E,t}) \\ &+ \quad E_t \sum_{t=0}^{\infty} \beta_E^t \lambda_{E,t}^2 [R_{L,t+1} L_t - \phi_t Q_{t+1} K_t] \end{aligned}$$

We have the set of the FOCs is:

$$K_t: \quad E_t\left(-\lambda^1_{E,t}Q_t + \beta_E\lambda^1_{E,t+1}R^K_{t+1} - \lambda^2_{E,t}\phi_t Q_{t+1}\right) = 0 \tag{62}$$

$$L_t: \quad E_t\left(\lambda^1_{E,t} - \beta_E\lambda^1_{E,t+1}R_{L,t+1} + \lambda^2_{E,t}R_{L,t+1}\right) = 0 \tag{63}$$

$$C_{E,t}: \quad U_{E,C}(C_E) = [C_{E,t}]^{-\gamma_E} = \lambda^1_{E,t} \tag{64}$$

From the FOCs above we derive the linearized Euler equation for the fully rational entrepreneur:

$$\begin{aligned}\tilde{C}_{E,t} &= \frac{\beta_E R^K - 1}{\beta_E\gamma_E(R^K-\phi)}(E_t[\tilde{Q}_{t+1}] - \tilde{Q}_t) - \frac{\beta_E R^K}{\beta_E\gamma_E(R^K-\phi)}E_t[\tilde{R}^K_{t+1}] \\ &+ \frac{\phi/R_L}{\beta_E\gamma_E(R^K-\phi)}E_t[\tilde{R}_{L,t+1}] + \frac{\beta_E R^K - 1}{\beta_E\gamma_E(R^K-\phi)}\tilde{\phi}_t + E_t[\tilde{C}_{E,t+1}]\end{aligned} \tag{65}$$

The behavioural entrepreneur :
The behavioural agent's budget constraint is:

$$L_t = [\bar{R_L} + m_{RL1}\hat{R}_{L,t}]L_{t-1} + [Q + m_{Q1}\hat{Q}_t]K_t - [R^K + m_{RK}\hat{R}^K_t][Q + m_{Q1}\hat{Q}_{t-1}]K_{t-1} + C_{E,t} - (T_E + m_T\hat{T}_{E,t}) \tag{66}$$

Similarly, the behavioural agent's budget constraint is:

$$E_t[R_L + m_{RL2}\hat{R}_{L,t+1}]L_t \leq (\phi + m_\phi\hat{\phi}_t)E_t[Q + m_{Q2}\hat{Q}_{t+1}]K_t \tag{67}$$

The value function of the optimization problem is

$$V_E(L_t, K_t) = max_{c_E}U_E(C_E) + \beta_E V_E(L_{t+1}, K_{t+1})$$

Similarly, we have Langragian function for the behavioural entrepreneur:

$$\begin{aligned}\mathbb{L}_E &= E_t\sum_{t=0}^{\infty}\beta^t_E U(C_{E,t}) \\ &+ E_t\sum_{t=0}^{\infty}\beta^t_E\lambda^1_{E,t}(L_t - ([\bar{R_L} + m_{RL1}\hat{R}_{L,t}]L_{t-1} + [Q + m_{Q1}\hat{Q}_t]K_t \\ &- [R^K + m_{RK}\hat{R}^K_t][Q + m_{Q1}\hat{Q}_{t-1}]K_{t-1} + C_{E,t} - (T_E + m_T\hat{T}_{E,t}))) \\ &+ E_t\sum_{t=0}^{\infty}\beta^t_E\lambda^2_{E,t}\left[E_t[R_L + m_{RL2}\hat{R}_{L,t+1}]L_t - \phi_t E_t[Q + m_{Q2}\hat{Q}_{t+1}]K_t\right]\end{aligned}$$

Similarly, at the optimum entrepreneur solves the following problem $V_E(L, K) = max_{C_E,L,K}\mathbb{L}$. The envelop theorem implies that

$$V_{E,R_{L,t+1}} = L_{E,R_{L,t+1}} = \beta^t_E\left[U_{E,R_{L,t+1}} + \beta_E\lambda^1_{E,t+1}L_t\right] \tag{68}$$

Notice that, the derivation above is on $R_{L,t+1}$ of the budget constraint. However, in the behavioural world, agent will pay a different level of attention on the future variable, say $R_{L,t+1}$ in the collateral constraint.

$$V_{E,E_t[R_{L,t+1}]} = L_{E,E_t[R_{L,t+1}]} = \beta_E^t \left[U_{E,R_{L,t+1}} + \lambda_{E,t}^2 L_t\right] \tag{69}$$

Where the shadow price on the collateral constraint, $\lambda_{E,t}^2 = \beta_E \lambda_{E,t+1}^1 - \frac{\lambda_{E,t}^1}{R_{L,t+1}}$, then we have:

$$V_{E,E_t[R_{L,t+1}]} = \beta_E^t \left[U_{E,R_{L,t+1}} + \beta_E \lambda_{E,t+1}^1 L_t - \frac{\lambda_{E,t}^1}{R_{L,t+1}} L_t\right] \tag{70}$$

Again applying the Envelop theorem to the remaining exogenous variables to the agent, R_{t+1}^K, Q_t, $E_t Q_{t+1}$, ϕ_t, and $T_{E,t}$ we have

$$V_{E,R_{t+1}^K} = L_{E,R_{t+1}^K} = \beta_E^t \left[U_{E,R_{t+1}^K} - \beta_E \lambda_{E,t+1}^1 Q_t K_t\right] \tag{71}$$

$$V_{E,Q_t} = L_{E,Q_t} = \beta_E^t \left[U_{E,Q_t} - \lambda_{E,t}^1 K_t + \beta_E \lambda_{E,t+1}^1 R_{t+1}^K K_t\right] \tag{72}$$

$$V_{E,E_tQ_{t+1}} = L_{E,E_tQ_{t+1}} = \beta_E^t \left[U_{E,E_tQ_{t+1}} - \lambda_{E,t}^2 \phi_t K_t\right] = \beta_E^t \left[U_{E,E_tQ_{t+1}} - \beta_E \lambda_{E,t+1}^1 \phi_t K_t + \frac{\lambda_{E,t}^1}{R_{L,t+1}} \phi_t K_t\right] \tag{73}$$

$$V_{E,\phi_t} = L_{E,\phi_t} = \beta_E^t \left[U_{E,\phi_t} - \lambda_{E,t}^2 Q_{t+1} K_t\right] = \beta_E^t \left[U_{E,\phi_t} - \beta_E \lambda_{E,t+1}^1 Q_{t+1} K_t + \frac{\lambda_{E,t}^1}{R_{L,t+1}} Q_{t+1} K_t\right] \tag{74}$$

$$V_{E,T_{E,t}} = L_{E,T_{E,t}} = \beta_E^t \left[U_{E,T_{E,t}} + \lambda_{E,t}^1\right] \tag{75}$$

Similarly, assuming at the default model we have the solution of the entrepreneur's consumption is

$$C_{E,t} = \Theta L_t$$

where $\Theta = \frac{(1-\beta_E)(R_L-\phi)}{\phi\beta_E}$.

By taking the derivative of $V_{E,R_{L,t+1}}$ w.r.t L_o we have the one-time change of $R_{L,t+1}$.

$$V_{E,R_L,L_o} = \beta_E^{t+1} \left[-\gamma_E \Theta L_o C_{E,o}^{-\gamma_E-1} + C_{E,o}^{-\gamma_E}\right] \tag{76}$$

Similarly, we derive the one-time change of the state variables derivation of objective function w.r.t the exogenous variables:

One-time change of $E_t R_{L,t+1}$.

$$V_{E,E_tR_L,L_o} = \beta_E^t \left[-\beta_E \gamma_E \Theta L_o C_{E,o}^{-\gamma_E - 1} + \beta_E C_{E,o}^{-\gamma_E} + \frac{\gamma_E \Theta L_o C_{E,o}^{-\gamma_E - 1}}{R_L} - \frac{C_{E,o}^{-\gamma_E - 1}}{R_L} \right] \tag{77}$$

Taking the derivative of V_{E,R_{t+1}^K} w.r.t K_o we have the one-time change of R_{t+1}^K.

$$V_{E,R_{t+1}^K,K_o} = L_{E,R_{t+1}^K,K_o} = \beta_E^{t+1} \left[\bar{Q} C_{E,o}^{-\gamma_E} \right] \tag{78}$$

Taking the derivative of V_{E,Q_t} w.r.t K_o we have the one-time change of Q_t.

$$V_{E,Q_t,K_o} = L_{E,Q_t,K_o} = \beta_E^t \left[-C_{E,o}^{-\gamma_E} + \beta_E C_{E,o}^{-\gamma_E} R^K \right] \tag{79}$$

Taking the derivative of $V_{E,E_tQ_{t+1}}$ w.r.t K_o we have the one-time change of $E_t Q_{t+1}$.

$$V_{E,E_tQ_{t+1},K_o} = L_{E,E_tQ_{t+1},K_o} = \beta_E^t \left[-\beta_E \phi C_{E,o}^{-\gamma_E} + \phi \frac{C_{E,o}^{-\gamma_E}}{R_L} \right] \tag{80}$$

Taking the derivative of V_{E,ϕ_t} w.r.t K_o we have the one-time change of ϕ_t.

$$V_{E,\phi_t,K_o} = L_{E,\phi_t,K_o} = \beta_E^t \left[-\beta_E Q C_{E,o}^{-\gamma_E} + Q \frac{C_{E,o}^{-\gamma_E}}{R_L} \right] \tag{81}$$

Taking the derivative of V_{E,T_t} w.r.t L_o we have the one-time change of T_t.

$$V_{E,T_t,L_o} = L_{E,T_t,L_o} = -\beta_E^t \gamma_E C_{E,o}^{-\gamma_E - 1} \tag{82}$$

Now, we apply the Benveniste - Sheinkman theorem for the entrepreneur's problem, we have:

$$\begin{pmatrix} V_L(L,K) \\ V_K(L,K) \end{pmatrix} = -U_{C_E}(C_E) \begin{pmatrix} \frac{\partial F/\partial L}{\partial F/\partial C_E} \\ \frac{\partial F/\partial K}{\partial F/\partial C_E} \end{pmatrix} \tag{83}$$

Where F(.) is the function containing two constraints of the entrepreneur.

$$\begin{pmatrix} \frac{\partial F/\partial L}{\partial F/\partial C_E} \\ \frac{\partial F/\partial K}{\partial F/\partial C_E} \end{pmatrix} = \begin{pmatrix} 1 - \frac{R_{L,t+1} Q_t}{\phi_t Q_{t+1}} \\ \frac{\phi_t Q_{t+1}}{R_{L,t+1}} - Q_t \end{pmatrix} = \begin{pmatrix} 1 - \frac{R_L}{\phi} \\ \left(\frac{\phi}{R_L} - 1 \right) Q \end{pmatrix} \tag{84}$$

Where the second equality shows we evaluate the partial change of the objective function at the steady state.

Hence, we have $V_L(L,K) = -U_{C_E}(C_E) \left(1 - \frac{R_L}{\phi} \right)$. By taking the total derivative of this expression w.r.t $R_{L,t+1}$ we get the partial derivative of consumption on the market variable or the loans return rate:

$$\partial_{\hat{R}_{L,t+1}} C_{E,o} = \frac{V_{E,R_{L,t},L_o}}{U_{E,C,C}} \left(\frac{\phi}{R_L - \phi}\right) \;=\; \beta_E^t \left(\frac{\phi}{R_L - \phi}\right)\left(\beta_E \Theta L_o - \beta_E \frac{C_{E,o}}{\gamma_E}\right) \tag{85}$$

Similarly, we can derive the partial change on consumption on other market exogenous variables:

$$\begin{aligned}\partial_{E_t \hat{R}_{L,t+1}} C_{E,o} &= \frac{V_{E,E_t R_{L,t},L_o}}{U_{E,C,C}} \left(\frac{\phi}{R_L - \phi}\right) \\ &= \beta_E^t \left(\frac{\phi}{R_L - \phi}\right)\left[-\beta_E \Theta L_o + \beta_E \frac{C_{E,o}}{\gamma_E} - \frac{\Theta}{R_L} L_o + \frac{C_{E,o}}{\gamma_E R_L}\right]\end{aligned} \tag{86}$$

$$\partial_{\hat{R}^K_{t+1}} C_{E,o} = \frac{V_{E,\hat{R}^K_{t+1},K_o}}{U_{E,C,C}} \left(\frac{R_L}{(R_L - \phi)Q}\right) \;=\; -\beta_E^t \left(\frac{R_L}{(R_L - \phi)}\right)\left(\frac{\beta_E C_{E,o}}{\gamma_E}\right) \tag{87}$$

$$\partial_{\hat{Q}_t} C_{E,o} = \frac{V_{E,\hat{Q}_t,K_o}}{U_{E,C,C}} \left(\frac{R_L}{(R_L - \phi)Q}\right) \;=\; \beta_E^t \left(\frac{R_L}{(R_L - \phi)Q}\right)\left(\frac{C_{E,o}}{\gamma_E} - \beta_E R^K \frac{C_{E,o}}{\gamma_E}\right) \tag{88}$$

$$\partial_{E_t \hat{Q}_{t+1}} C_{E,o} = \frac{V_{E,E_t \hat{Q}_{t+1},K_o}}{U_{E,C,C}} \left(\frac{R_L}{(R_L - \phi)Q}\right) \;=\; \beta_E^t \left(\frac{R_L}{(R_L - \phi)Q}\right)\left(\frac{\beta_E \phi C_{E,o}}{\gamma_E} - \frac{\phi C_{E,o}}{\gamma_E R_L}\right) \tag{89}$$

$$\partial_{\hat{\phi}_t} C_{E,o} = \frac{V_{E,\hat{\phi}_t,K_o}}{U_{E,C,C}} \left(\frac{R_L}{(R_L - \phi)Q}\right) \;=\; \beta_E^t \left(\frac{R_L}{(R_L - \phi)}\right)\left(\beta_E \frac{C_{E,o}}{\gamma_E} - \frac{C_{E,o}}{\gamma_E R_L}\right) \tag{90}$$

$$\partial_{\hat{T}_t} C_{E,o} = \frac{V_{E,\hat{T}_t,K_o}}{U_{E,C,C}} \left(\frac{\phi}{R_L - \phi}\right) \;=\; \beta_E^t \left(\frac{\phi}{(R_L - \phi)}\right)\Theta \tag{91}$$

So that we have the Taylor Expansion for $C_{E,t}$, or the behavioural entreprenur's consumption function:

$$\begin{aligned}\hat{C}_{E,t} &= E_t^{BR} \sum_{\tau \geq t} \partial_{\hat{R}_{L,t+1}} C_{E,o} \hat{R}_{L,\tau+1} + E_t^{BR} \sum_{\tau \geq t} \partial_{E_t \hat{R}_{L,t+1}} C_{E,o} \hat{R}_{L,\tau+1} + E_t^{BR} \sum_{\tau \geq t} \partial_{E_t \hat{R}^K_{t+1}} C_{E,o} \hat{R}^K_{\tau+1} \\ &+ E_t^{BR} \sum_{\tau \geq t} \partial_{\hat{Q}_t} C_{E,o} \hat{Q}_\tau + E_t^{BR} \sum_{\tau \geq t} \partial_{E_t \hat{Q}_{t+1}} C_{E,o} \hat{Q}_{\tau+1} + E_t^{BR} \sum_{\tau \geq t} \partial_{\hat{\phi}_t} C_{E,o} \hat{\phi}_\tau\end{aligned}$$

$$+ \quad E_t^{BR} \sum_{\tau \geq t} \partial_{\hat{T}_t} C_{E,o} \hat{T}_\tau \tag{92}$$

We have $E_t(X_{t+1}) = m_X E_t^{BR}(X_{t+1})$, where m_X is the agent's inattention degree on the market variable X. So that from the fully rational consumption function (92) we can derive the boundedly rational entrepreneur's consumption function nested in a fully rational expectation as follows:

$$\begin{aligned}
\hat{C}_{E,t} &= E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{\hat{R}_{L,t+1}} C_{E,o} \right)_{L_o = 0} m_{RL1} \hat{R}_{L,\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{E_t \hat{R}_{L,t+1}} C_{E,o} \right)_{L_o = 0} m_{RL2} \hat{R}_{L,\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{E_t \hat{R}^K_{t+1}} C_{E,o} \right) m_{RK} \hat{R}^K_{\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{\hat{Q}_t} C_{E,o} \right) m_{Q1} \hat{Q}_\tau \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{E_t \hat{Q}_{t+1}} C_{E,o} \right) m_{Q2} \hat{Q}_{\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{\hat{\phi}_t} C_{E,o} \right) \hat{\phi}_\tau \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \left(\partial_{\hat{T}_t} C_{E,o} \right) m_T \hat{T}_\tau
\end{aligned} \tag{93}$$

Where $\bar{m}$ is the agent's myopia parameter of the future state. By substituting the partial changes of consumption w.r.t market variables into the equation (93) we have the following consumption function:

$$\begin{aligned}
\hat{C}_{E,t} &= E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{\phi}{R_L - \phi} \right) \left(-\frac{\beta_E C_{E,o}}{\gamma_E} \right) m_{RL1} \hat{R}_{L,\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{\phi}{R_L - \phi} \right) \left(\beta_E \frac{C_{E,o}}{\gamma_E} + \frac{C_{E,o}}{\gamma_E R_L} \right) m_{RL2} \hat{R}_{L,\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{R_L}{(R_L - \phi)} \right) \left(-\frac{\beta_E C_{E,o}}{\gamma_E} \right) m_{RK} \hat{R}^K_{\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{R_L}{(R_L - \phi) Q} \right) \left(\frac{C_{E,o}}{\gamma_E} - \beta_E R^K \frac{C_{E,o}}{\gamma_E} \right) m_{Q1} \hat{Q}_\tau \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{R_L}{(R_L - \phi) Q} \right) \left(-\frac{\beta_E \phi C_{E,o}}{\gamma_E} + \frac{\phi C_{E,o}}{\gamma_E R_L} \right) m_{Q2} \hat{Q}_{\tau+1} \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{R_L}{(R_L - \phi)} \right) \left(\beta_E \frac{C_{E,o}}{\gamma_E} - \frac{C_{E,o}}{\gamma_E R_L} \right) \hat{\phi}_\tau \\
&+ E_t \sum_{\tau \geq t} \bar{m}^{\tau - t} \beta_E^{\tau - t + 1} \left(\frac{\phi}{(R_L - \phi)} \right) \Theta m_T \hat{T}_\tau
\end{aligned} \tag{94}$$

By expanding and simplifying the consumption equation (94). Then dividing both sides by $C_{E,o}$, we have the linearized version of the behavioural entrepreneur's consumption function as in equation (17).

A.3 Firms - Deviation of the non-zero steady state inflation Phillips Curve

The homogeneous production technology in the economy is:

$$Y_t = A_t H_t^{\alpha} \tag{95}$$

There is a probability of $1-\xi$ at each period that the price of each retail good i is set optimally to $P_t^0(i)$; otherwise it is held fixed.
Retail behavioural producer i, given the common real marginal cost $MC_t(i) = MC_t$ chooses $\{P_t^0(i)\}$ to maximize discounted real profits

$$\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(i) \left[P_t^0(i) - P_{t+k} MC_{t+k} \right] \tag{96}$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor over the interval $[t, t+k]$, subject to

$$Y_{t+k}(i) = \left(\frac{P_t^O(i)}{P_{t+k}} \right)^{-\zeta} Y_{t+k} \tag{97}$$

The solution to this is

$$\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} Y_{t+k}(i) \left[\frac{P_t^0(i)}{P_{t+k}} - \frac{1}{(1-1/\zeta)} MC_{t+k} \right] = 0 \tag{98}$$

which leads to

$$\frac{P_t^0(m)}{P_t} = \frac{1}{1-1/\zeta} \frac{\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta-1} Y_{t+k}} \tag{99}$$

where k periods ahead inflation is defined by

$$\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t} = \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} \cdot \cdot \frac{P_{t+k}}{P_{t+k-1}} = \Pi_{t+1} \Pi_{t+2} \cdot \cdot \Pi_{t+k}$$

Note that $\Pi_{t,t+1} = \Pi_{t+1}$ and $\Pi_{t,t} = 1$.
Let us define

$$J_t = \frac{1}{1-\frac{1}{\zeta}} \mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\zeta} Y_{t+k} MC_{t+k}$$

$$= \frac{1}{1-\frac{1}{\zeta}} Y_t MC_t + \xi \mathbb{E}_t^{BR} \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta} J_{t+1} \tag{100}$$

$$JJ_t = \mathbb{E}_t^{BR} \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \Pi_{t,t+k}^{\zeta-1} Y_{t+k}$$

$$= Y_t + \xi \mathbb{E}_t^{BR} \Lambda_{t,t+1} \Pi_{t,t+1}^{\zeta-1} JJ_{t+1} \tag{101}$$

Then (99) can be written as

$$\frac{P_t^0(m)}{P_t} = \frac{J_t}{JJ_t} \tag{102}$$

By the law of large numbers the evolution of the price index is given by

$$P_{t+1}^{1-\zeta} = \xi P_t^{1-\zeta} + (1-\xi)(P_{t+1}^0)^{1-\zeta} \tag{103}$$

which can be written as

$$1 = \xi \Pi_t^{\zeta-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{1-\zeta} \tag{104}$$

We first transform the equations (100), (101) to the expectations of the **behavioural** agents, where we also utilise the relation $\mathbb{E}_t^{BR}\Lambda_{t,t+k} = \frac{1}{\mathbb{E}_t^{BR} R_{t+1,t+k}} = \frac{1}{\mathbb{E}_t^{BR}\left[\frac{R_{n,t,t+k-1}}{\Pi_{t+1,t+k}}\right]}$, and employing the assumption about the firms' myopia about the future state such that: $\mathbb{E}_t^{BR}(X_{t+1} - X) = \bar{m}_f \mathbb{E}_t(X_{t+1} - X)$. In addition, firms are inattentive to the market's variables which are exogenous to them. Hence, we can re-write the equations (100) and (101) in a recursive form as follows:

$$J_t = \frac{1}{1-\frac{1}{\zeta}}(Y_t)(MC + m_{fmc}\hat{MC}_t) + \xi \mathbb{E}_t \frac{(\Pi + \bar{m}_f m_{f\pi} \hat{\Pi}_{t+1})^{\zeta}}{\mathbb{E}_t\left(\frac{R_n + m_{fr}\hat{R}_{n,t}}{\Pi + \bar{m}_f m_{f\pi}\hat{\Pi}_{t+1}}\right)} (J + \bar{m}_f \hat{J}_{t+1}) \tag{105}$$

$$JJ_t = Y_t + \xi \mathbb{E}_t \frac{(\Pi + \bar{m}_f m_{f\pi} \hat{\Pi}_{t+1})^{\zeta-1}}{\mathbb{E}_t\left(\frac{R_n + m_{fr}\hat{R}_{n,t}}{\Pi + \bar{m}_f m_{f\pi}\hat{\Pi}_{t+1}}\right)} (JJ + \bar{m}_f \hat{J}J_{t+1}) \tag{106}$$

B Steady State

Notice that, the method we combine the non-rational elements into the model makes the agents still fully rational for the steady state variables. It is only their sensitivity to deviations from the deterministic steady state that is partially myopic.

$$R = \frac{1}{\beta} \tag{107}$$

$$\frac{J}{JJ} = \left(\frac{1-\xi\Pi^{\zeta-1}}{1-\xi}\right)^{\frac{1}{1-\zeta}} \tag{108}$$

$$MC = \left(1 - \frac{1}{\zeta}\right) \frac{J}{JJ} \frac{1-\xi\beta\Pi^{\zeta}}{1-\xi\beta\Pi^{\zeta-1}} \tag{109}$$

$$\Delta = \frac{(1-\xi)\left(\frac{J}{JJ}\right)^{-\zeta}}{1-\xi\Pi^{\zeta}} \tag{110}$$

$$PWP = MC \tag{111}$$

$$KYW = \frac{PWP * \alpha}{RK - 1 + \delta} \tag{112}$$

$$IY = KYW * \delta * \Delta \tag{113}$$

$$CY = 1 - IY - gy - CeY \tag{114}$$

$$N = \left(\frac{(1-\alpha)\Delta MC}{CY}\right)^{\frac{1}{1+\sigma}} \tag{115}$$

$$K = \left[\frac{R^K - 1 + \delta}{\alpha * MC * A * N^{1-\alpha}}\right]^{\frac{1}{1+\alpha}} \tag{116}$$

$$Y^W = AN^{1-\alpha}K^{\alpha} \tag{117}$$

$$Y = \frac{Y^W}{\Delta} \tag{118}$$

$$G = gy * Y \tag{119}$$

$$T = G - T_E \tag{120}$$

$$I = IY * Y \tag{121}$$

$$W = \frac{(1-\alpha)PWPYW}{N} \tag{122}$$

$$C = Y - G - I - C_E \tag{123}$$

$$J = \frac{YMCMS}{(1-\frac{1}{\zeta})(1-\xi\beta\Pi^{\zeta})} \tag{124}$$

$$JJ = \frac{Y}{(1-\xi\beta\Pi^{\zeta-1})} \tag{125}$$

$$U = \frac{C^{1-\gamma}}{1-\gamma} - \frac{H^{1+\sigma}}{1+\sigma} \tag{126}$$

$$V = \frac{U}{1-\beta} \tag{127}$$

Entrepreneur's steady state

$$CeY = 0.1 \tag{128}$$

$$Spread = 0.01/4 \tag{129}$$

$$Q = 1 \tag{130}$$

$$R_L = \frac{R_L^n}{\Pi} \tag{131}$$

$$R_L^n = \frac{\zeta_L}{\zeta_L - 1} R^n \tag{132}$$

$$R^K = \frac{1}{\beta_E} + \phi - \frac{\phi}{R_L \beta_E} \tag{133}$$

$$L = \frac{\phi K}{R_L} \tag{134}$$

$$C_E/Y = (R_K - \phi)(1 - \beta_E)K/Y + T_E/Y \tag{135}$$

If we calibrate the steady state of financial spread in equation (129) we can solve for the value of the elasticity on the loan market, ζ_L. Similarly, to solve for the value of T_E in the steady state we calibrate the proportion value of entrepreneur's consumption on output as in equation (128).

C Equilibrium

$$\tilde{C}_t = \frac{\bar{m}}{R-(R-1)m_Y} E_t[\tilde{C}_{t+1}] - \frac{m_r}{\gamma(R-(R-1)m_Y)} E_t[\tilde{R}_{t+1}] \tag{136}$$

$$\tilde{W}_t = \gamma \tilde{C}_t + \sigma \tilde{N}_t \tag{137}$$

$$\tilde{I}_t = \frac{1}{1+\beta}\tilde{I}_{t-1} + \frac{\beta}{1+\beta}\tilde{I}_{t+1} + \frac{1}{(1+\beta)\phi_I}\tilde{Q}_t \tag{138}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1} + \delta \tilde{I}_t \tag{139}$$

$$\begin{aligned}
\tilde{C}_{E,t} &= \frac{\beta_E \phi(m_{RL2} - m_{RL1}) + (\phi/R_L)m_{RL2}}{\gamma_E(R^K - \phi)[1-(1-\beta_E)m_T]} E_t[\tilde{R}_{L,t+1}] \\
&- \frac{\beta_E R^K m_{RK}}{\gamma_E(R^K - \phi)[1-(1-\beta_E)m_T]} E_t[\tilde{R}^K_{t+1}] \\
&- \frac{(\beta_E R^K - 1)m_{Q1}}{\gamma_E(R^K - \phi)[1-(1-\beta_E)m_T]} \tilde{Q}_t \\
&+ \frac{(\beta_E R^K - 1)m_{Q2}}{\gamma_E(R^K - \phi)[1-(1-\beta_E)m_T]} E_t\tilde{Q}_{t+1} \\
&+ \frac{(\beta_E R^K - 1)}{\gamma_E(R^K - \phi)[1-(1-\beta_E)m_T]} \tilde{\phi}_t \\
&+ \frac{\beta_E \bar{m}}{1-(1-\beta_E)m_T} E_t[\tilde{C}_{E,t+1}]
\end{aligned} \tag{140}$$

$$\tilde{L}_t = \tilde{K}_t + m_{Q2}\tilde{Q}_{t+1} + m_\phi \tilde{\phi}_t - m_{RL2}\tilde{R}_{L,t+1} \tag{141}$$

$$\begin{aligned}
\tilde{L}_t &= R_L \tilde{L}_{t-1} + R_L m_{RL1}\tilde{R}_{L,t} + \frac{R_L}{\phi}[\tilde{K}_t + m_{Q1}\tilde{Q}_t - R^K \tilde{K}_{t-1} - R^K m_{RK}\tilde{R}^K_t - R^K m_{Q1}\tilde{Q}_{t-1}] \\
&+ \left(\frac{C_E}{L} - \frac{T_E}{L} m_T\right)\tilde{C}_{E,t}
\end{aligned} \tag{142}$$

$$\tilde{J}_t = (1-\beta\xi\Pi^\zeta)\left(\tilde{Y}_t + m_{fmc}\tilde{MC}_t\right) \tag{143}$$

$$
\begin{aligned}
& + \quad \beta\xi\Pi^{\zeta}\mathbb{E}_t\left((1+\zeta)\bar{m}_f m_{f\pi}\tilde{\Pi}_{t+1} - m_{fr}\tilde{Rn}_t + \bar{m}_f\tilde{J}_{t+1}\right) && (144)\\
\tilde{JJ}_t &= (1-\beta\xi\Pi^{\zeta-1})\tilde{Y}_t && (145)\\
& + \quad \beta\xi\Pi^{\zeta-1}\mathbb{E}_t\left(\zeta\bar{m}_f m_{f\pi}\tilde{\Pi}_{t+1} - m_{fr}\tilde{Rn}_t + \bar{m}_f\tilde{JJ}_{t+1}\right) && (146)\\
\tilde{\Pi}_t &= \frac{1-\xi\Pi^{\zeta-1}}{\xi\Pi^{\zeta-1}}(\tilde{J}_t - \tilde{JJ}_t) && (147)\\
\tilde{\Pi}_t &= \frac{1-\xi\Pi^{\zeta-1}}{\xi\Pi^{\zeta-1}}(\tilde{J}_t - \tilde{JJ}_t) && (148)\\
\tilde{MC}_t &= \tilde{W}_t + \tilde{N}_t - \tilde{Y}^w_t + ms_t && (149)\\
\tilde{R}^K_t &= \left(1-\frac{1-\delta}{R^K}\right)(\tilde{MC}_t + \tilde{Y}^w_t - \tilde{K}_{t-1}) + \frac{1-\delta}{R^K}\tilde{Q}_t - \tilde{Q}_{t-1} && (150)\\
\tilde{Y}^w_t &= \tilde{A}_t + (1-\alpha)\tilde{N}_t + \alpha\tilde{K}_{t-1} && (151)\\
\tilde{\Delta}_t &= \tilde{Y}^w_t - \tilde{Y}_t && (152)\\
\tilde{\Delta}_t &= \xi\Pi^{\zeta}\left(\tilde{\Delta}_{t-1} + \zeta\tilde{\Pi}_t\right) - \zeta(1-\xi\Pi^{\zeta})(\tilde{J}_t - \tilde{JJ}_t) && (153)\\
\tilde{Y}_t &= \frac{C}{Y}\tilde{C}_t + \frac{C_E}{Y}\tilde{C}_{E,t} + \frac{I}{Y}\tilde{I}_t + \frac{G}{Y}\tilde{G}_t && (154)\\
\tilde{R}_t &= R^n_{t-1} - \tilde{\Pi}_t && (155)\\
\tilde{R}_{L,t} &= R^n_{L,t-1} - \tilde{\Pi}_t && (156)\\
R^n_{L,t} &= R^n_t && (157)\\
R^n_t &= \rho^{rn}R^n_{t-1} + (1-\rho^{rn})(\theta^{\pi}\tilde{\Pi}_t + \theta^{dy}(\tilde{Y}_t - \tilde{Y}_{t-1})) + \epsilon^r_t && (158)\\
\tilde{\phi}_t &= \theta_l(\tilde{L}_t - \tilde{L}_{t-1}) + \epsilon^{\phi}_t && (159)\\
\tilde{G}_t &= \rho_G\tilde{G}_{t-1} + \epsilon^G_t && (160)\\
\tilde{A}_t &= \rho_A\tilde{A}_{t-1} + \epsilon^A_t && (161)\\
\tilde{ms}_t &= \rho_{ms}\tilde{ms}_{t-1} + \epsilon^{ms}_t && (162)
\end{aligned}
$$

D Linear quadratic problem

Optimal monetary and macroprudential policy will be studied relying on the society's quadratic loss function. We assume that the CB does not have the full knowledge about the actual behavioural of the economy, so that they use the fully rational expectation system to rank welfare of different regimes.

We first summarize the non-linear equilibrium of the fully rational expectation.

$$
\begin{aligned}
\mathbb{E}_t\Lambda_{t+1} &= \beta\mathbb{E}_t\frac{C_t}{C_{t+1}} && (163)\\
1 &= \mathbb{E}_t[R_{t+1}\Lambda_{t+1}] && (164)\\
R_t &= \frac{R_{n,t-1}}{\Pi_t} && (165)\\
W_t &= \kappa H^{\sigma}_t C_t && (166)
\end{aligned}
$$

Price setting:

$$JJ_t = Y_t + \xi\mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta-1}JJ_{t+1}] \tag{167}$$

$$J_t = \left(\frac{1}{1-\frac{1}{\zeta}}\right)Y_tMC_tMS_t + \xi\mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta}J_{t+1}] \tag{168}$$

$$1 = \xi\Pi_t^{\zeta-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{1-\zeta} \tag{169}$$

$$\Delta_t = \xi\Pi_t^{\zeta}\Delta_{t-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{-\zeta} \tag{170}$$

$$Y_t^W = (A_tH_t)^{\alpha}(K_{t-1})^{1-\alpha} \tag{171}$$

$$W_t = \alpha MC_t\frac{Y_t^W}{H_t} \tag{172}$$

$$Y_t = \frac{Y_t^W}{\Delta_t} \tag{173}$$

Capital sector:

$$K_t = (1-\delta)K_{t-1} + (1-S(X_t))I_tIS_t \tag{174}$$

$$X_t = \frac{I_t}{I_{t-1}} \tag{175}$$

$$S(X_t) = \phi_X(X_t-1)^2 \tag{176}$$

$$S'(X_t) = 2\phi_X(X_t-1) \tag{177}$$

$$R_t^K = \frac{\frac{(1-\alpha)MC_tY_t^W}{K_{t-1}} + Q_t(1-\delta)}{Q_{t-1}} \tag{178}$$

$$\begin{aligned} 1 &= Q_t\left[1-S(X_t)-S'(X_t)X_t\right]IS_t \\ &+ \mathbb{E}_t\left[\Lambda_{t+1}Q_{t+1}S'(X_{t+1})X_{t+1}^2IS_{t+1}\right] \end{aligned} \tag{179}$$

Banking sector:

$$L_t = R_{L,t}L_{t-1} + Q_tK_t - R_t^KQ_{t-1}K_{t-1} + C_{E,t} - T_{E,t} \tag{180}$$

$$\mathbb{E}_t[R_{L,t+1}]L_t = \phi_tE_t[Q_{t+1}]K_t \tag{181}$$

$$0 = \Lambda_{E,t}^1Q_t - \beta_E\mathbb{E}_t\left[\Lambda_{E,t+1}^1R_{t+1}^K\right]Q_t + \Lambda_{E,t}^2\phi_t\mathbb{E}_t[Q_{t+1}] \tag{182}$$

$$0 = -\Lambda_{E,t}^1 + \beta_E\mathbb{E}_t\left[\Lambda_{E,t+1}^1R_{L,t+1}\right] - \Lambda_{E,t}^2\mathbb{E}_t[R_{L,t+1}] \tag{183}$$

$$\frac{1}{C_{E,t}} = -\Lambda_{E,t}^1 \tag{184}$$

$$R_{L,t} = \frac{R_{n,t-1}}{\Pi_t} \tag{185}$$

$$R_{spread,t} = \frac{R_t^K}{R_t} \tag{186}$$

$$TE_t = TE \tag{187}$$

Aggregate market clearing condition:

$$Y_t = C_t + G_t + I_T + CE_t \tag{188}$$

The system above has 26 equations along with 27 endogenous variables: $C_t, H_t, \Lambda_t, R_t, R_{n,t}, \Pi_t, W_t, J_t, JJ_t, Y_t, Y_t^w, \Delta_t, MC_t, K_t, S_t, X_t, I_t, S_t^{'}, R_t^K, R_{L,t}, Q_t, CE_t, L_t, R_{L,t}, TE_t, \Lambda_{E,t}^1, \Lambda_{E,t}^2$. When we consider the social planer's problem the additional equation will come in a form of an optimization problem of the social planner's objective given the system above. We define the social planer's objective function as a composite of the Household's and Enterprise's welfare function. One-period utility functions of Household and Enterprise are defined as follows:

$$U_t = log(C_t) - \kappa \frac{H_t^{\sigma+1}}{\sigma+1} \tag{189}$$

$$UE_t = log(CE_t) \tag{190}$$

Let's assume that social planner assigns weights on household's and enterprise's utilities proportionally to the steady state consumption levels:

$$U_t^{planner} = w_1 U_t + w_2 UE_t \tag{191}$$

Where $w_1 = \frac{C}{C+CE}$ and $w_2 = \frac{CE}{C+CE}$ Hence, the social planner's welfare function is as follows:

$$Wel_t = \sum_{\tau=0}^{\infty} \beta U_{t+\tau}^{planner} = U_t^{planner} + \beta Wel_{t+1} \tag{192}$$

E Estimation Details

E.1 Identification

Following Iskrev and Ratto (2010), we provide the identification (locally) analysis of the our tool model here. In the upper panel of the figure the bars depict the identification strength of the parameters based in the Fisher information matrix normalized by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (red bars). Intuitively, the bars represent the normalized curvature of the log likelihood function at the prior mean in the direction of the parameter. If the strength is 0 (for both bars) the parameter is not identified as the likelihood function is flat in this direction. The larger the absolute value if the bars, the stronger the identification. Hence, it is clear that all parameters are identified in the model.

E.2 MCMC Convergence

The convergence property is represented in figure (5). The appended (Interval) shows the Brooks and Gelman's convergence diagnostics for the 80% interval. The blue line shows the 80% interval/quantile range based on the pooled draws from all sequences, while the

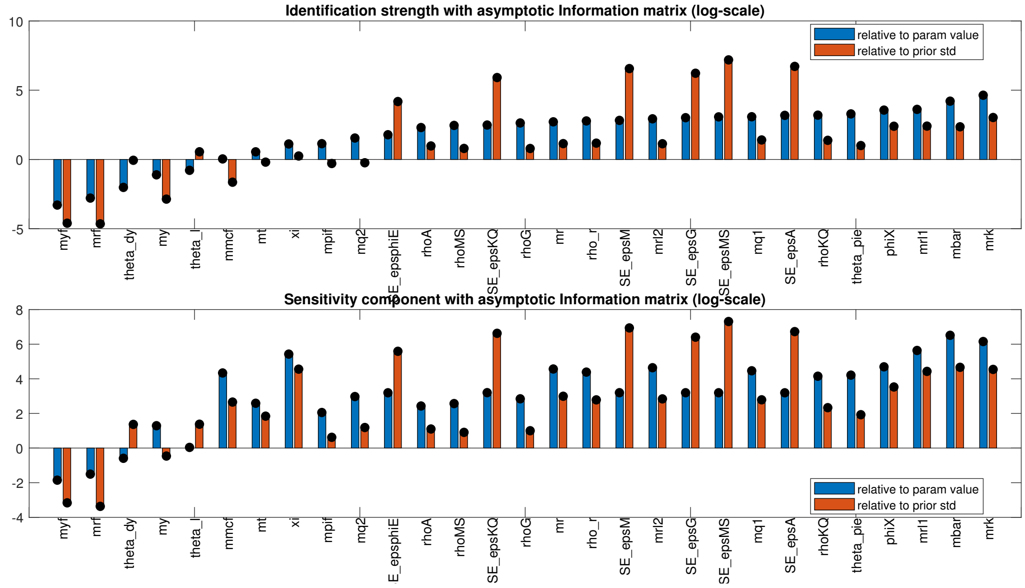

Figure 4: Identification Strength in the tool Model

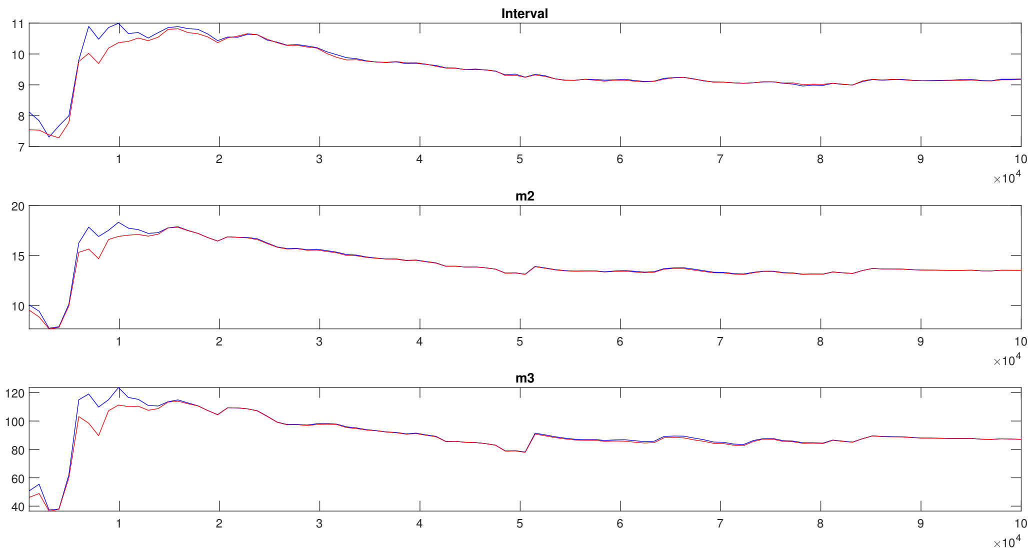

Figure 5: Multivariate convergence diagnostic

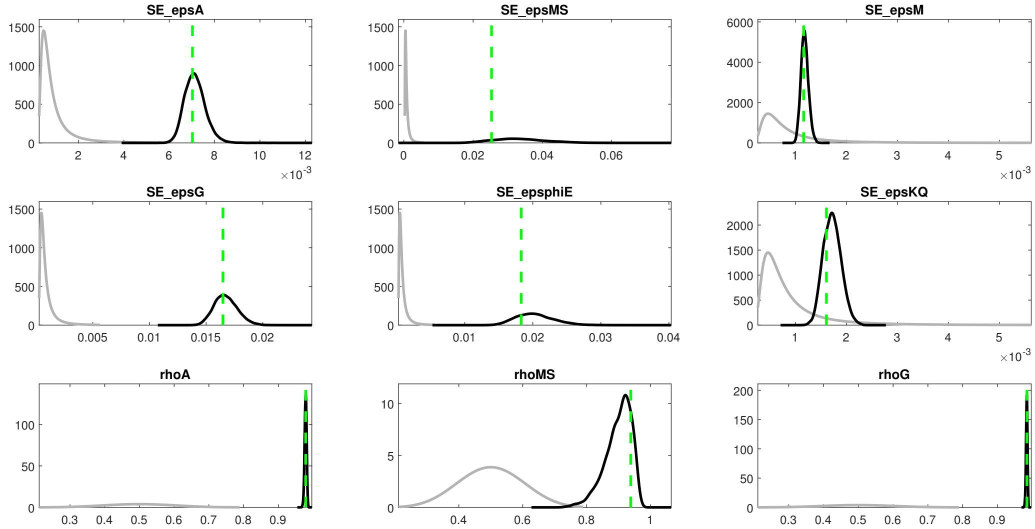

Figure 6: Priors and Posteriors for 100000 MCMC draws

red line shows the mean interval range based on the draws of the individual sequences. The appended (m2) and (m3) show an estimate of the same statistics for the second and third central moments, i.e. the squared and cubed absolute deviations from the pooled and the within-sample mean, respectively. All statistics are based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters. Convergence is indicated by the two lines stabilizing and being close to each other.

The figures from (6) to (9) indicate the prior-posterior plots. The grey line shows the prior density, while the black line shows the density of the posterior distribution. The green horizontal line indicates the posterior mode. If the posterior looks like the prior, either your prior was a very accurate reflection of the information in the data or the parameter under consideration is only weakly identified and the data does not provide much information to update the prior.

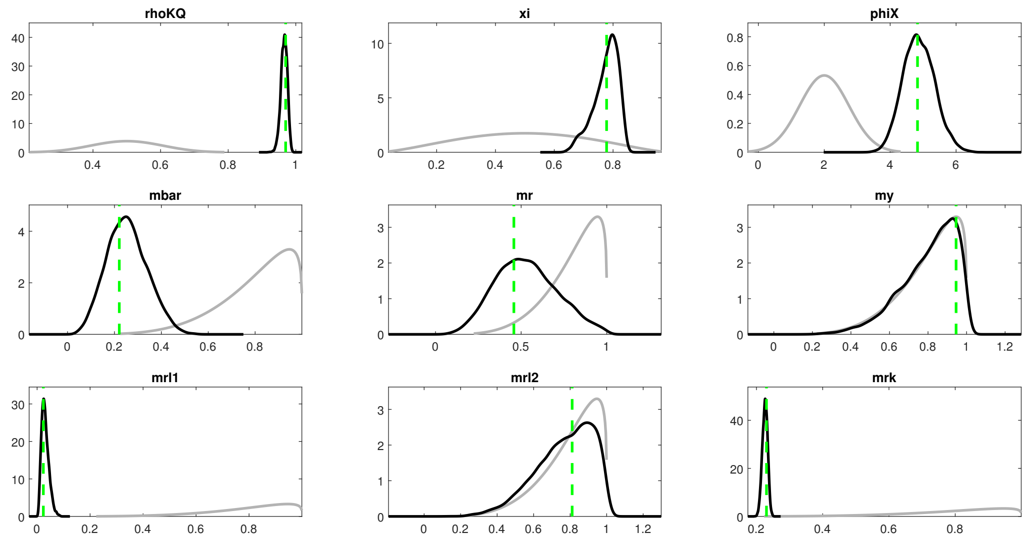

Figure 7: Priors and Posteriors for 100000 MCMC draws

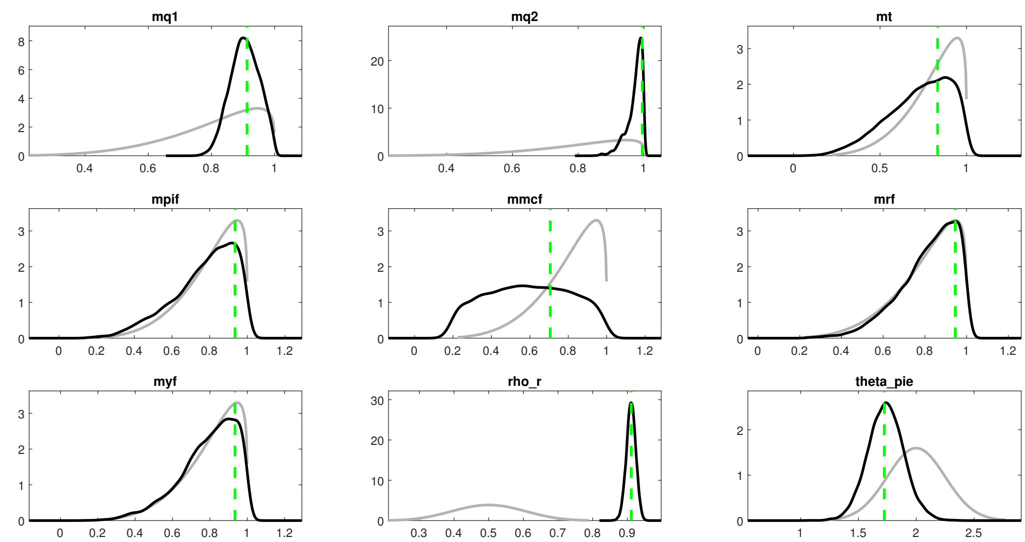

Figure 8: Priors and Posteriors for 100000 MCMC draws

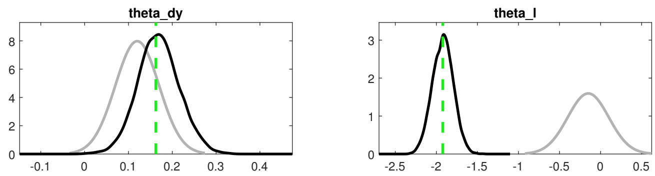

Figure 9: Priors and Posteriors for 100000 MCMC draws